

## Problem Set #4

1. Suppose the production possibility frontier, *PPF*, is given by:

$$PPF: Y = 100 - \frac{x^2}{100}$$

and the marginal rate of transformation is:

$$MRT = \frac{x}{50}$$

The only two consumers, A and B, in the economy have the following utility functions:

$$U_A = x_A y_A \quad U_B = x_B + y_B$$

- a) To attain Pareto optimality, what is the total quantity of each good that must be produced? (These quantities **will** be produced if  $P = MC$  for each).
- b) If person A has all of good  $x$  and person B has all of good  $y$  initially, what will be the final distribution of goods in a price system?
2. A competitive industry faces the following market demand function:

$$Q^D = 1,200 - 4P \quad , \text{ where } Q \text{ is the total industry output.}$$

The 100 identical firms in the industry face the following short-run costs:

$$SRTC = 1 + 2.5q^2$$
$$SRMC = 5q \quad , \text{ where } q \text{ is the individual firm's output.}$$

- a)
- i) What is the short run supply function for a firm?
  - ii) What is the short run market supply function?
  - iii) What is the short run equilibrium price and quantity in the market?
  - iv) What is the short run equilibrium output for each firm?
  - v) What is the profit for each firm?

- b) Suppose the firm finds itself in Stage I long run equilibrium making a profit. Graph the situation for the firm and the industry.
- i) Suppose a production tax of \$5 per unit is imposed on the industry. What will happen to the price and output of each firm?
  - ii) What happens to the profit-level of each firm? Carefully explain your answer.
  - iii) Suppose the firm had been in Stage II long run equilibrium before the tax was imposed. What will happen to the Stage II long run supply curve and the number of firms in the industry after the tax is imposed?
  - iv) Who bears the tax burden ultimately?

3. A monopolist publisher is selling a magazine in two markets: through student subscriptions and newsstands. The demand curves in the two markets are as follows:

$$\begin{array}{ll} Q_1 = 1,000 - 20P_1 & \text{students' demand} \\ Q_2 = 1,400 - 40P_2 & \text{newsstands' demand} \end{array}$$

- a) Find the marginal revenue functions in each market. When the publisher is maximizing profits with a different price in each market, which market will yield the higher marginal revenues, or will marginal revenues be the same in each market?
- b) The publisher's cost function in the short run is:

$$SRTC = 100 + 10Q \quad , \text{ where } Q = Q_1 = Q_2$$

Find the quantity sold in each market and the price in each market. What are the respective values of marginal revenue?

- c) Find the profits of the firm. What conditions must hold to make this price discrimination possible?
- d) Suppose the Justice Department issues an order requiring the firm to charge the same price to all consumers. Now what is the profit maximizing output level, price, and profits for the publisher?

4. A monopolistic steel producer has the following cost function:

$$C(q) = q^2 + 60q + 100$$

The demand curve the firm faces is:

$$P = 200 - q$$

Note: If Total Cost is  $TC = q^2 + kq + m$ , then  $MC = 2q + k$ .

For example, if  $C(q) = q^2 + 60q + 100$  then  $MC = 2q + 60$ .

Unfortunately, for every unit of steel produced 0.1 units of pollution,  $z$ , are released into the adjacent river (i.e.  $z = 0.1q$ ). The cost of cleaning up pollution is known to be:

$$\text{Total Pollution Costs} = 100 + 400z, \text{ where } z \text{ is the amount of pollution.}$$

- a) What is the price and output level of the producer if she can pollute freely?
- b) Suppose the producer is forced to internalize the externality. That is, she is forced to pay pollution costs. Now what is the price and output level of the producer?
- c) Will this scheme eliminate pollution? How much pollution will be produced in the two situations?
- d) Suppose the government wants to reduce the production of pollution to the same extent as achieved in (b) but with a per-unit production tax. What level should the tax be?