Problem Set #5

1. Consider a market in which two firms produce a homogenous product. Market demand is given as:

\[ P = 100 - 0.5(q_a + q_b) \]

where \( q_a \) and \( q_b \) are the levels of output of firm A and firm B respectively.

The cost functions of each firm are also given as:

\[
\begin{align*}
TC_a &= 5q_a \\
MC_a &= 5 \\
TC_b &= 0.5q_b^2 \\
MC_b &= q_b
\end{align*}
\]

a) What is the Pareto optimal market price (\( P \)) and output level of each firm (\( q_a \) and \( q_b \)), and what would be the profit that each firm makes at the Pareto Optimal price and output levels?

b) Suppose that these two firms recognize their mutual interdependence and agree to act in unison in order to maximize the total profit of the industry. Both output levels are then under a single control and the industry is, in effect, a monopoly. What are \( q_a \) and \( q_b \), market price (\( P \)) and profit of each firm?

c) Now consider the Cournot solution where each firm maximizes its profits on the assumption that the quantity produced by its rival is invariant with respect to its own quantity decision. That is, each firm maximizes its own profit assuming the output of the other firm is given. Again calculate the market price, output and profit for each firm.

d) Compare total output, price and total profit in (a) with (b) and (c). Do they make sense?
2. Assume that the market for hamburgers is competitive in a particular area. At the competitive price for hamburgers, there is a demand for hamburger cooks:

\[ L = 168 - 12w \]

, where \( w \) is the wage paid to hamburger cooks.

The supply of hamburger cooks in the area depends on the wage rate that is offered to them:

\[ L = -24 + 6w \]

a) Graph the demand and supply for hamburger cooks.

b) Assume that this labor market is competitive. Find the equilibrium wage and employment for hamburger cooks.

c) Now suppose that all the hamburger stores in the area form a coalition. The market for hamburgers is still competitive since people can drive to other areas to get hamburgers. But the market for hamburger cooks is now monopsonistic, since these cooks are mostly high-school students who do not have licenses to drive elsewhere for work. Find the new equilibrium wage and employment.