Due in class on Sept 18. Bring a copy of your answers to use in class discussion.

1. Consider a firm that faces a downward-sloping demand:

\[ P = 100 - Q \]

and whose total cost function is given by:

\[ TC = 1000 + 30Q \]

a. Graph the demand curve, marginal cost curve, and average cost curve.

b. Is the firm a natural monopoly?

c. Locate the first-best outcome on your graph and label it F. Calculate consumer surplus and profit if the firm were to produce at F. Is this outcome feasible for the firm?

d. Now locate the second-best outcome and label it S. (Hint: if you find more than one candidate answer, the correct one is the one with the greatest consumer surplus.)

e. Suppose the firm acted as an unregulated monopolist. Calculate the profit-maximizing output and price. Locate the point on the graph and label it M.

f. Derive the expression for the price elasticity of demand, \( \varepsilon \), in this market. (Hint: \( \varepsilon = -\left( \frac{P}{Q} \right)\frac{dQ}{dP} \). Verify that the unregulated monopolist does not produce in the inelastic portion of demand. (Reminder: demand is inelastic when \( \varepsilon \) is below 1 in magnitude.)

g. Summarize your results in a table showing price, output, consumer surplus, profits, and total surplus for each of the three points F, S, and M. Verify that these results have the properties discussed in class: (i) Is total surplus greatest at the first-best outcome? (ii) Is profit greatest at M? (iii) Does the second best outcome give the firm zero profits? (iv) Moving from M to S to F, does price decrease and output increase?

2. For each of the situations below, draw a graph like that in Figure 1.15, page 42, of the text. Include all the elements that are in Fig. 1.15, but make sure that they are positioned in relation to each other as required by the situation. If the situation is impossible, explain why.

a. The regulated firm chooses more output, less labor, and more capital than if unregulated.

b. The regulated firm chooses less output, more labor, and more capital than if unregulated.

c. The regulated firm chooses more output, more labor, and more capital than if unregulated.

d. The regulated firm chooses less output, less labor, and less capital than if unregulated.
3. Consider a firm whose production function is such that it has the following expansion path.

Suppose the firm is subject to ROR regulation with \( f > r \). Will the regulated firm choose a higher or lower \( K/L \) ratio that if it were unregulated? (Note: We know that the regulated firm will choose a \( K/L \) ratio that is too high for its output level, but here we are asking a different question: we are asking whether the \( K/L \) ratio is higher than if the firm were not regulated, given this expansion path.)

4. Consider a firm under ROR regulation with \( f > r \). Explain in words why lowering the allowed rate of return causes the regulated firm to increase its use of capital.

5. In class we assumed that the firm maximizes profits. Suppose a firm maximizes revenues rather than profits. To be more precise, suppose that the firm wants to make as much revenue as possible without losing money (maximizes revenue subject to profits not being negative.) Let’s see the impact of ROR regulation on this firm.

   a. On a K-L graph, draw a zero profit contour and an expansion path. Label the second-best outcome as \( S \). Let’s assume that this point is in the inelastic portion of demand.
   b. Without regulation, this firm will choose the level of output at which \( MR = 0 \) (i.e., where the elasticity of demand is –1.) Explain why this is so.
   c. On the graph from (a), draw the isoquant for the output level that the firm chooses when unregulated (i.e., the isoquant for the output level at which \( MR = 0 \).) Be sure to place this isoquant on the graph appropriately in relation to \( S \).
   d. We now want to know what input combination will this unregulated firm will choose. Show that the firm’s input choice is indeterminant within a range of possible input combinations. Show on your graph the input combinations that the firm might choose.
   e. Suppose now that the firm is placed under ROR regulation with \( f > r \). Show that the regulation will NEVER (i.e, under no circumstances) induce the firm to increase its
output beyond what it chose without regulation. (You’ll need to add the constraint curve to show this.)

f. Show that the regulation will NEVER induce the firm to produce with a more efficient K/L ratio. Draw a new graph (since the old one is getting crowded) and show on this graph the input combinations that the regulated firm might choose.

g. Suppose that the firm, when unregulated, happened to choose the most efficient K/L ratio for its level of output (i.e., was on the expansion path.) Show that the ROR regulation will, in this case, necessarily induce the firm to move to an inefficient K/L ratio.

6. In class we discussed the A-J effect for a firm that faces a constant cost of capital, r. This problem considers the case where the firm faces an upward sloping supply curve for capital, such that the cost of capital rises as the firm uses more capital. Suppose the cost of capital is given by \( r = a + mK \), where \( a > 0 \) and \( m > 0 \). The regulator behaves just the same as before, choosing an \( f \) such that the monopolists’ profits can be no greater than \((f-r)K\).

   a. Why might a firm in the real world face an upward sloping supply curve for capital? (This is a question out of your micro class.)

   b. Draw a profit hill silhouette, like that in Figure 1.13, page 39 of the text.

   c. Given the supply curve above, derive the constraint on profits that the firm faces.

   d. Draw this constraint on a graph of profits and K. (This graph is analogous to Figure 1.13, page 39, in the text, except that the constraint plane will have a different shape because of the rising cost of capital.)

   e. Using two separate graphs, show that it is possible, depending on the shape of the constraint plane in relation to the profit hill, for the firm to choose either (i) a point on the profit hill or (ii) a point in the interior of the profit hill?

   f. Show in case (ii) – where the firm chooses an interior point – that the firm has an incentive to waste.

7. “The regulator should not set \( f = r \) because that would imply that the firm makes zero profit and a firm which makes zero profit has no incentive to produce any particular quantity, let alone produce efficiently.” Reconcile this statement with the case of perfect competition, where firms make zero profit but still always produce a unique, well-defined amount and produce that amount efficiently.

8. In two handwritten pages, summarize the results of McFarland and Keeler/Ying, and describe why the results matter.