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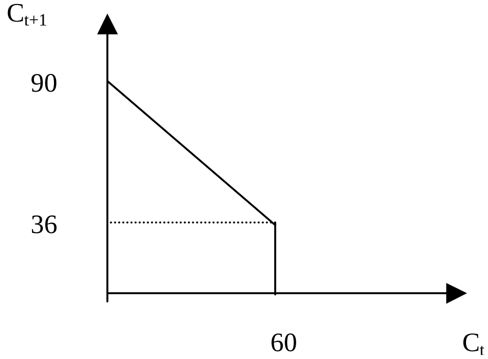
Economics 136
Solutions to Problem Set #1

Problem (1):

a. Crusoe cannot consume more than 60 units in the current period. For every unit that Crusoe stores he can consume 9/10 of that unit next period (in addition to his next-period endowment of 36). Thus, we can write Crusoe's budget constraint as $C_{t+1} = 36 + 9/10(60 - C_t) = 90 - 9/10C_t$, for $C_t \leq 60$. Of course, if he consumes exactly 60 today he can consume any amount up to 36 tomorrow. We can use these facts to draw his budget line as follows:



b. There are numerous ways to set up this problem: you can use the method of Lagrange multipliers; you can substitute the budget constraint into the utility function and solve as an unconstrained optimization problem; you could try to calculate the MRS and set it equal to the slope of the budget line. All of these methods are equivalent. I will use the last method. First, let us look for an interior solution, one not at the end points or "kink" in the budget line.

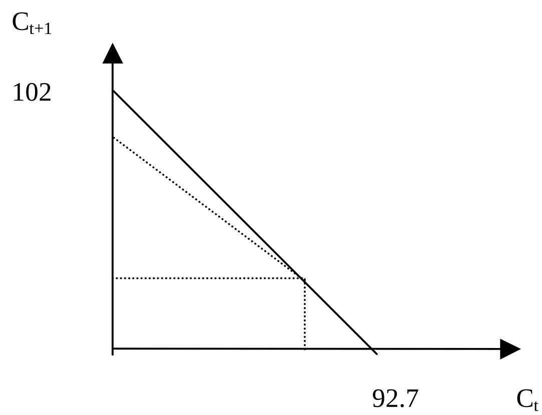
$$MRS = -\frac{\partial U / \partial C_t}{\partial U / \partial C_{t+1}} = -\frac{1/C_t}{\beta / C_{t+1}} = -\frac{C_{t+1}}{9/11C_t} = -\frac{9}{10} = \text{slope of budget line}$$
$$\Rightarrow C_{t+1} = \frac{9}{10} \cdot \frac{9}{11} C_t.$$

Substituting this last expression into the equation for the budget line we found in part(a) above, we get

$$\frac{9}{10} \cdot \frac{9}{11} C_t = 90 - \frac{9}{10} C_t \Rightarrow \frac{9}{10} C_t \left(1 + \frac{9}{11}\right) = 90 \Rightarrow \frac{9}{10} \cdot \frac{20}{11} C_t = 90 \Rightarrow \frac{18}{11} C_t = 90$$
$$\Rightarrow C_t^* = 90 \cdot \frac{11}{18} = 55 \text{ and } C_{t+1}^* = \frac{9}{10} \cdot \frac{9}{11} \cdot 55 = 40.5$$

Note that these consumptions levels imply that savings is 5 and utility is 7.036.

c. Crusoe's budget line now becomes $C_{t+1} = 36-(1.1)(60-C_t) = 102-1.1C_t$, and is graphed as follows:



It is clear from the graph that ability to trade across time has greatly increased the choices Crusoe has available to him. This will generally make him better off.

d. **(i)** As we did in part(b), let's set Crusoe's MRS = to the slope of his budget line, and then substitute into the budget constraint:

$$MRS = \frac{C_{t+1}}{9/11C_t} = \frac{11}{10} = \text{slope of budget line} \Rightarrow C_{t+1} = \frac{9}{10}C_t$$
$$\Rightarrow .9C_t = 102-1.1C_t \Rightarrow 2C_t = 102 \Rightarrow C_t^* = 51 \Rightarrow C_{t+1}^* = 45.9$$

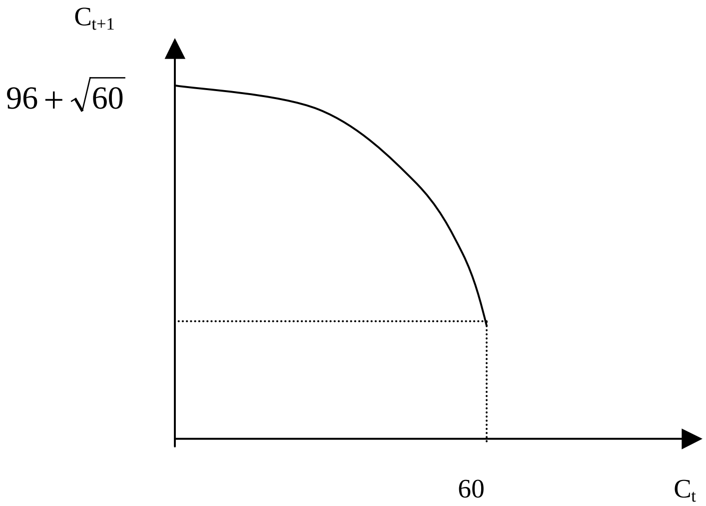
(ii) Now utility is 7.063, which is indeed greater than what we found in part (b).

(iii) Savings is now 9 instead of 5 as before. Introducing a financial market had the same effect as raising the interest rate here. (This should be clear from the graph in part (c); also you can think of the case where only storage was available as offering a negative return on savings). Since savings increased when we "raised" the interest rate, the substitution effect dominates.

e. Friday's budget constrains it $C_{t+1} = 54+(1.1)(40-C_t) = 98-1.1C_t$. Since Friday has the same preferences as Crusoe he has the same MRS too. Thus we know that Friday will consume in the same proportions as Crusoe--that is, he wants $C_{t+1} = 9/10C_t$ also. Substituting this into Friday's budget constraint gives $.9C_t = 98-1.1C_t \Rightarrow 2C_t = 98 \Rightarrow C_t^* = 49$ and $C_{t+1}^* = 44.1$. Hence, Friday wants to borrow 9 coconuts, which is exactly what Crusoe wants to lend. The credit market for coconuts clears, which shows 10% is an equilibrium interest rate as required.

f. To derive the consumption possibilities frontier (CPF), think about how future consumption depends on current consumption. Future consumption depends on how many coconuts Crusoe plants, his capital. Specifically, $C_{t+1} = 36+k+\sqrt{k}$. The amount of capital depends in turn on how many coconuts Crusoe consumes today--namely, $k = 60-C_t$. Hence, his CPF is given by

$C_{t+1} = 36 + (60 - C_t) + \sqrt{60 - C_t} = 90 - C_t + \sqrt{60 - C_t}$, for $C_t \leq 60$. It is graphed below.



g. (i) To find how many coconuts Crusoe wants to plant, set the slope of the CPF (which in this case is just - MPK) equal to -(1+r). That is, we solve

$$-\left(1 + \frac{1}{2\sqrt{k}}\right) = -1.1 \Rightarrow \sqrt{k} = \frac{1}{.2} = 5 \Rightarrow k = 25.$$

(ii). After planting 25 coconuts, Crusoe has 35 left today and will have 36+25+5=66 tomorrow. Thus his budget constraint is $C_{t+1} = 66 + (1.1)(35 - C_t) = 104.5 - 1.1C_t$. Crusoe still has the same preferences as before, so he will set C_{t+1}=.9C_t. So we have

$$.9C = 104.5 - 1.1C_t \Rightarrow 2C_t = 104.5 \Rightarrow C_t^* = 52.25 \text{ and } C_{t+1}^* = 47.025$$

(iii) The above calculation implies that Crusoe wants to borrow 17.25 coconuts. To summarize, Crusoe borrows 17.25 coconuts, giving him a total of 77.25 today. Of these, he plants 25 and consumes 52.25. Next period he has 66 coconuts, 47.025 of which he'll eat and the remaining 18.975 will be repaid to his creditor.

Problem 2:

a.

E(X) = (0.1)(40) +(0.2)(21)+ (0.4)(12)+ (0.2)(-16)+ (0.1)(-30) = 6.8
E(Y) = (0.1)(-50) +(0.2)(70)+ (0.4)(30)+ (0.2)(-80)+ (0.1)(40) = 9.0
Var(X) = $(0.1)(40-6.8)^2$ +$(0.2)(21-6.8)^2$+ $(0.4)(12-6.8)^2$+ $(0.2)(-16-6.8)^2$+ $(0.1)(-30-6.8)^2$ = 400.76
Var(Y) = $(0.1)(-50-9)^2$ +$(0.2)(70-9)^2$+ $(0.4)(30-9)^2$+ $(0.2)(-80-9)^2$+ $(0.1)(40-9)^2$ = 2949
Cov(X,Y) = (0.1)(40-6.8)(-50-9)+(0.2)(21-6.8) (70-9)+ (0.4)(12-6.8))(30-9)+ (0.2)(-16-6.8)(-80-9)+ (0.1)(-30-6.8))(40-9)=312.8

b.

E(U) = E(0.5X+1) = 0.5E(X) + 1=0.5*6.8+1 = 4.4
Var(U) = Var(0.5X+1) = 0.5^2Var(X) = 100.19

E(Z) = E(2Y-2) = 2E(Y)-2 = 2*9-2= 16
Var(Z) = Var(2Y-2) = 2^2Var(Y) = 1251.2

c.

$E(\omega X+(1-\omega)Y) = \omega E(X)+(1-\omega)E(Y)$

$Var((\omega X+(1-\omega)Y) = \omega^2 Var(X)+(1-\omega)^2 Var(Y)+2\omega(1-\omega)Cov(X,Y)$

ω = 0.5

E(.5X+.5Y) = 7.9

Var(.5X+.5Y)=993.44

ω = 0.25

E(.25X+.75Y) =8.45

Var(.25X+.75Y)= 1800.9

d.

To solve for the minimum-variance portfolio weight, you can set up

$$\underset{w}{Min} \quad Var(Z) = w^2 Var(X) + (1-w)^2 Var(Y) + 2w(1-w)\operatorname{cov}(X,Y)$$

Then,

$$\frac{\partial Var(Z)}{\partial w} = 2wVar(X) + 2(1-w)(-1)Var(Y) + (2-4w)\operatorname{cov}(X,Y)$$

$$\frac{\partial Var(Z)}{\partial w} = 0 \quad \Rightarrow \quad 2w\{Var(X)+Var(Y)-2Cov(X,Y)\} = 2Var(Y)-2Cov(X,Y)$$

$$w* = \frac{Var(Y)-Cov(X,Y)}{Var(X)+Var(Y)-2Cov(X,Y)}$$

Then,

$$\varpi^* = \frac{2949-312.8}{400.76+2949-2\times 312.8} = 0.9677$$