Department of Economics
University of California, Berkeley

Spring, 2000
Prof. Pierce

Economics 136
Problem Set #3
(Due in Lecture Tuesday, March 7th)

1. a. ***Paul Samuelson's (1965) Formula for Stock Prices***

(i) Suppose that you are trying to value a stock that will pay dividends only in the next two periods, $t+1$ and $t+2$. Beginning in period three, the stock will pay no dividends and will be worthless. Using the formula,

$$P_t = \frac{E(P_{t+1} + d_{t+1})}{1+\rho}$$

find a formula for P_{t+1} in terms of d_{t+2} and P_{t+2}.

(ii) Show that $P_{t+2} = 0$.

(iii) Use this result to find a formula for P_t in terms of d_{t+1} and d_{t+2}.

(iv) Now, suppose that the stock pays dividends indefinitely far into the future. By repeated substitution for P_{t+1}, P_{t+2}, P_{t+3}, etc., find a formula for P_t in terms of d_{t+1}, d_{t+2}, d_{t+3}, … *Note:* to be precise, you must assume that the expected present value of the "final" dividend (received infinitely far in the future) is zero, that is,

$$\lim_{j\to\infty} \frac{E(P_{t+j})}{(1+\rho)^j}$$

Hint: you may be able to derive the formula by repeating steps (i), (ii), and (iii) for a stock that pays no dividends (and becomes worthless) starting in period four. By doing so, you should see a pattern emerging…

b. Consider again the stock that pays dividends only in periods $t+1$ and $t+2$ (and becomes worthless thereafter). Suppose that instead of paying the dividend d_{t+1} in period $t+1$, the firm invests the funds in an asset, "*Z*", that has a "normal" return of ρ each period. Thus, $d_{t+1} = 0$. In period $t+2$, the firm pays as dividends both its regular dividend, d_{t+2}, and the proceeds from its sale of asset "*Z*". How is the current price of the firm's stock, P_t, affected? Why?

2. Suppose you are the CEO of GM, and yesterday's price of GM stock was \$60 per share. This morning the Japanese government announced that they found some serious defects in imported GM automobiles, which is expected to hurt next period's earning of GM.

(a) If investors on Wall Street expect that price of GM stock will be \$58.24 with probability of 0.5 or \$54.08 with probability of 0.5 in a year from today, what will today's price of GM be? (Assume the current one-year discount rate for a stock of GM's risk class is $\rho = 8$ %, and GM does not plan to pay a dividend next year.)

(b) Now you, as the CEO of the company, are worried about losing your job because of unhappy shareholders. You think you need to do something to please your shareholders. But luckily you learned in Econ 136 when you were an undergraduate at Cal that the current stock price is the discounted expected value of the sum of next year's dividend and price (thus,

$P_t = \frac{E_t(P_{t+1} + d_{t+1})}{1+\rho}$). Therefore, you decided to pay a dividend next year to restore the current value of the stock. How much would the dividend have to be?

(c) There is a slight problem, though. Where does the money to pay the dividend come from? Right away you figure out that you can borrow the funds from banks, and because of GM's good credit standing there is no problem borrowing enough for a dividend. You plan to borrow from the bank next year and pay back the loan a year after. Now how will Wall Street react to your decision if stock markets are efficient? (Hint: What will happen to GM's earnings when they pay back what they have borrowed? Precisely, will the expected price of the stock next year be affected by the fact that GM has to pay back its loan?

3. Given the following spot rates (yields to maturity) for zero coupon bonds,

y_1 = 5% (one year bond)
y_2 = 6% (two year bond)
y_3 = 7% (three year bond)
y_4 = 6% (four year bond)

suppose the expectation hypothesis holds and calculate:

(a) $E(r_2)$ (the expected one year spot rate for period 2)
(b) $E(r_3)$ (the expected one year spot rate for period 3)
(c) $E(r_4)$ (the expected one year spot rate for period 4)

4. Suppose the current 1-year interest rate is r_1 = 6% and the current forward rate for the second year is f_2 = 7% and the current forward rate for the third year is f_3 = 8%.

(a) Construct the yield curve.
(b) Compute the expected two-year interest rate starting one year from today under the Expectations hypothesis. (Note this is not an *annualized* rate.)
(c) Suppose instead that you suspect that investors demand a liquidity premium to compensate for risk. Now what information does the yield curve give you about expected future one-year interest rates?