\( L = \bar{L} \), the full-employment supply of labor. Then the dynamic equations of the model can be written as:

\[
\dot{q} - rq - AF_K(K, \bar{L}) - \frac{(q - 1)^2}{2\chi},
\]

\( K = \left( \frac{q - 1}{\chi} \right) K. \)

The steady state of the model occurs where \( \dot{q} = 1 \) and \( AF_K(K, \bar{L}) = r \).

Imagine a phase diagram with \( q \) on the vertical axis and \( K \) on the horizontal axis. The locus along which \( \dot{K} = 0 \) is horizontal at \( \dot{q} = 1 \), with \( K \) rising above it and falling below. On the other hand, consider the slope of the schedule along which \( \dot{q} = 0 \). It is given by

\[
\frac{dq}{dK} \Big|_{\dot{q}=0} = \frac{\chi AF_KK}{\chi r - (q - 1)}.
\]

At \( q = 1 \) (the steady state) the slope is negative, but as \( q \) rises the slope becomes positive. To the right of this schedule \( q \) is rising and to the left \( q \) is falling. We get a saddlepath adjustment to the steady state, with \( q \) falling as \( K \) rises.

What does \( q \) represent? The general solution to a differential equation such as (2) is a forward-looking integral expression (as you can verify by differentiating).\(^1\) (All of this is true even if we allow the interest rate \( r \) to vary over time.)

The general solution (for a constant interest rate) is

\[
q(t) = \int_t^\infty e^{-rt} \left[ AF_K(K(s), L(s)) + \frac{\chi}{2} \left( \frac{I(s)}{K(s)} \right)^2 \right] ds + be^{rt}.
\]

where I have made the substitution

\[
\frac{(q - 1)^2}{2\chi} = \frac{\chi}{2} \left( \frac{I}{K} \right)^2
\]

and \( b \) is an arbitrary constant. The economically relevant solution imposes the transversality condition

\[
\lim_{t \to \infty} e^{-rt}q(t)K(t) = 0
\]

which obliges us to set \( b = 0 \) above. In that case

\[
q(t) = \int_t^\infty e^{-rt} \left[ AF_K(K(s), L(s)) + \frac{\chi}{2} \left( \frac{I(s)}{K(s)} \right)^2 \right] ds,
\]

\(^1\)Use the following fact from calculus. Let

\[
f(t) = \int_{a(t)}^{b(t)} g(s, t) ds.
\]

Then

\[
f'(t) = \frac{\partial g(s, t)}{\partial t} ds.
\]

\[e^{rt}g(b(t), t) + \int_{a(t)}^{b(t)} \frac{\partial g(s, t)}{\partial t} ds.
\]
which means that the shadow value of a unit of installed capital equals the discounted future marginal products, plus the future contributions to lowering the installation costs of optimal investments (it is cheaper at the margin to add capital to a larger pre-existing capital stock).

The $q$ variable defined above is *marginal* $q$, the shadow value of an *extra* unit of capital, given $K$. Empirical work on investment, however, does not have access to this variable: researchers must use as a proxy stock-market value divided by total capital-in-place, $\Pi/K$, which amounts to *average* $q$. What is the relationship between average and marginal $q$? This was clarified in a famous 1982 article in *Econometrica* by Fumio Hayashi.

Notice that

$$\frac{d(qK)}{dt} = q\dot{K} + \dot{q}K$$

$$= rqK - \left( AF_K K + \frac{\chi I^2}{2K} \right) + qI$$

$$= rqK - \left( AF(K, L) - wL + \frac{\chi I^2}{2K} \right) + I \left( 1 + \frac{I}{K} \right)$$

$$= r (qK) - \left[ AF(K, L) - wL - I - \frac{\chi I^2}{2K} \right].$$

Imposing the transversality condition, we can integrate forward (solving for the composite variable $qK$) to conclude that

$$q(t)K(t) = \int_t^\infty e^{-r(s-t)} [A(s)F(K(s), L(s)) - w(s)L(s) - I(s) - \frac{\chi}{2}(I(s)^2/K(s))] ds = \Pi(t).$$

We see that marginal $q$ and average $q$ are equal:

$$q = \frac{\Pi}{K}.$$

The key facts used to derive this prediction are that markets are competitive and that the function $\psi(K, L, I) = F(K, L) - \frac{\chi I^2}{2K}$ displays constant returns to scale, i.e., for any nonnegative number $\lambda$, $\psi(\lambda K, \lambda L, \lambda I) = \lambda \psi(K, L, I)$. 

We see that marginal $q$ and average $q$ are equal:

$$q = \frac{\Pi}{K}.$$