Economics 202A Lecture Outline, November 29, 2007 (Version 2.0)

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Issues in Monetary Policy

In this lecture I survey several issues important in the design and implementation of monetary policy in practice. Some of these are related to the government's revenue needs, as discussed in the last lecture, but we also go beyond that question to consider other problems.

Money and welfare: Milton Friedman's "optimum quantity of money"

A useful dynamic framework for thinking about monetary policy in a world of *flexible* prices was provided by Miguel Sidrauski and William Brock.[1] The representative consumer maximizes

$$\int_t^{\infty} u\left[c(s), m(s)\right] \mathrm{e}^{-\delta(s-t)} \mathrm{d}s,$$

where c is consumption and $m \equiv M/P$ the stock of real balances held. Above, δ is the individual's subjective rate of discount, which can differ from the real interest rate. We are motivating a demand for money by assuming that the individual derives a flow of utility from his/her holdings of real balances — implicitly, these help the person economize on transaction costs, provide liquidity, etc.

Total real financial assets a are the sum of real money m and real bonds b, which pay a real rate of interest $r(t)$ at time t:

$$a = m + b.$$

[1] See Miguel Sidrauski, "Rational Choice and Patterns of growth in a Monetary Economy," *American Economic Review* 57 (May 1967): 534-44; and William A. Brock, "Money and Growth: The Case of Long Run perfect Foresight," *International Economic Review* 15 (October 1974): 750-77.

Let $\tau(t)$ be a transfer that the individual receives from the government each instant.[2] Then if we assume an endowment economy with output $y(t)$, the evolution of wealth is given by the differential equation

$$\begin{aligned}\dot{a} &= y + rb + \tau - c - \pi m \\ &= y + ra + \tau - c - (r + \pi)m.\end{aligned}$$

Since this last constraint incorporates the portfolio constraint that $a = m + b$, we need no longer worry about it. Under an assumption that we have perfect foresight, so that actual $\pi = \dot{P}/P$ equals expected inflation, the Fisher equation tells us that the nominal interest rate is

$$i = r + \pi,$$

so the last constraint becomes

$$\dot{a} = y + ra + \tau - c - im.$$

We can analyze the individual optimum using the Maximum Principle. If λ denotes the costate variable, the (current-value) Hamiltonian is

$$H = u\left(c, m\right) + \lambda\left(y + ra + \tau - im\right).$$

In the maximization problem starting at time t, $a(t)$ is predetermined at the level of the individual, who chooses optimal paths for c and m. The Pontryagin necessary conditions are

$$\frac{\partial H}{\partial c} = u_c - \lambda = 0,$$

$$\frac{\partial H}{\partial m} = u_m - \lambda i = 0,$$

$$\dot{\lambda} - \delta\lambda = -\frac{\partial H}{\partial a} = -\lambda r.$$

[2] Beware: in the last lecture the same symbol τ denoted taxes, or negative transfers, so all signs preceding τ are reversed in this lecture compared to the last.

To make life simple, let us assume that the cross-derivative $u_{cm} = 0$ — making $u(c, m)$ additively separable in consumption and real balances[3]. Then we can rewrite the last equation as

$$u_{cc}\dot{c} = u_c(\delta - r)$$

or equivalently, as

$$\frac{\dot{c}}{c} = -\frac{u_c}{cu_{cc}}(r - \delta).$$

Do you recognize this as the continuous-time version of the intertemporal bond Euler equation? For the isoelastic utility function with intertemporal substitution elasticity σ, we write this as

$$\frac{\dot{c}}{c} = \sigma(r - \delta),$$

an equation you will see *ad nauseam* in Economics 202B.[4]

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[3]This means that the utility function takes the form

$$u(c, m) = \upsilon(c) + \nu(m)$$

for strictly concave functions $\upsilon(c)$ and $\nu(m)$. Below, we will sometimes write the marginal utilities $u_m(c, m)$ and $u_c(c, m)$ in their general forms, as functions of c and m, respectively, even though that dependence is trivial when $u_{cm} = 0$.

[4]To see that this is actually a familiar equation, let us imagine that we have a time interval of length h, that the gross return to lending over that period is $1 + rh$, and that the discount factor between periods is $\beta = (1 + \delta h)^{-1}$. Then the Euler equation (in the isoelastic case, for example) would be

$$c_t^{-\frac{1}{\sigma}} = \frac{1 + rh}{1 + \delta h} c_{t+h}^{-\frac{1}{\sigma}}.$$

Take logs of both sides and divide by h to get

$$\frac{\log c_{t+h} - \log c_t}{h} = \frac{\sigma[\log(1 + rh) - \log(1 + \delta h)]}{h}.$$

As h gets small, the approximations $\log(1+rh) \approx rh$ and $\log(1+\delta h) \approx \delta h$ bcome arbitrarily close, and therefore so does the approximation

$$\frac{\log c_{t+h} - \log c_t}{h} \approx \sigma(r - \delta).$$

Let us consider the model's equilibrium next. The simplest assumption is that output is constant at level y, so that, in equilibrium $\dot{c}/c = 0$ and therefore $r = \delta$.

What is the equilibrium rate of inflation? We will assume that the government prints money to make transfers, and in such a way as to maintain a constant growth rate μ of the money supply. We therefore are assuming that real transfers are given by

$$\tau = \frac{\dot{M}}{P} = \left(\frac{\dot{M}}{M}\right)\frac{M}{P} = \mu m.$$

Important point: Individuals know this rule (under rational expectations), but they interpret it as $\tau = \mu \underline{m}$, where $\underline{m}$ denotes the economy's *aggregate per capita* real balances. As an individual, you are under no obligation to choose your own real balances m to equal $\underline{m}$. Thus, you will take $\underline{m}$ to be an exogenously given datum in solving your own optimization problem. That is exactly how we set up the preceding individual optimization problem — with τ being exogenous to the individual. It is only *in equilibrium* that the condition $m = \underline{m}$ must hold (because we have a representative-agent economy). So we are allowed to impose the government budget constraint $\tau = \mu m$ only when we solve for the equilibrium *after* having derived the individual's money and consumption demands. (Similarly, if we imposed the equilibrium condition $c = y$ prior to deriving the individual's first order conditions, we would never be able to conclude that $r = \delta$ in an equilibrium with constant output. The reason $r = \delta$ in equilibrium is that only that level of the real interest rate makes people choose $c = y$ as their optimum consumption level.)

Since we have a representative-agent economy (and have abstracted from government debt), equilibrium bond holdings are $b = 0$. If we substitute the other equilibrium conditions $c = y$ and $\tau = \mu m$ into the individual constraint

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Letting $h \to 0$, we therefore obtain

$$\frac{\dot{c}}{c} = \sigma\left(r - \delta\right)$$

in the limit of continuous time.

$\dot{a} = y + ra + \tau - c - im$, we get

$$\dot{m} = (\mu - \pi)m$$

By combining the first-order conditions for c and m, we obtain the money-demand relationship

$$\frac{u_m(c,m)}{u_c(c,m)} = i = r + \pi,$$

so substitution yields the equilibrium relationship

$$\dot{m} = \left[\mu + r - \frac{u_m(y,m)}{u_c(y,m)}\right] m.$$

Under our assumption that $u_{cm} = 0$, this equation yields unstable dynamics unless $\dot{m} = 0$ and $\pi = \mu$. Thus, it is an equilibrium for the inflation rate to be constant and equal to the constant growth rate of the money supply. In that case, the nominal interest rate is $i = r + \mu$.

Welfare and the zero lower bound: As usual, the demand for money falls when the nominal interest rate rises. But notice something important. It costs nothing (in principle) for the government to "produce" money, yet money yields utility. On narrow welfare grounds, therefore, it might seem best for the government to set a "price" for monetary services equal to their marginal cost of zero.

If the nominal interest rate i is positive, however, the fact that the government is printing money imposes a tax on real balances, as we have seen. Perhaps the resulting tax revenue is funding some useful government spending, perhaps not. But in any case, it might well be better to tax things other than real balances. Milton Friedman argued, on these grounds, that the "optimum" growth rate for the money supply is in fact $-r$, a shrinkage rate equal to the real interest rate, which makes the nominal interest rate equal to zero! In this case, if there is a "satiation" level of the money supply m_s such that $u_m(y, m_s) = 0$, and remains at zero for higher real-balance levels, then people will hold real balances of at least m_s. They will not be led to economize on a service that it costs society nothing to produce.

The zero lower bound and the liquidity trap

An interesting point about Friedman's optimum is that $i = 0$ is also the lower-bound on the nominal interest rate. At this point, the real return on money ($-\pi = r$) equals the real return r on bonds; but if the (negative) inflation rate were to fall further, no one would be willing to hold bonds. That is because the real return on money would be $-\pi > r$, whereas the real return on bonds would be the lower number $i - \pi = r$. Alternatively, money would have a nominal rate of return of precisely zero, whereas bonds would fall in nominal value at the rate $-i$.

To better understand the implications of the zero lower bound on i, let us first examine the behavior of budget constraints in Friedman's optimum quantity of money (OQM) equilibrium. As per our analysis last time, the individual's lifetime constraint looks like this:

$$m(t) + b(t) = \int_t^\infty \mathrm{e}^{-r(s-t)} \left[c(s) + i(s)m(s) - y(s) - \tau(s)\right] \mathrm{d}s.$$

In an equilibrium with $b = 0$, $i = 0$, and $c = y$, we get

$$m(t) = -\int_t^\infty \mathrm{e}^{-r(s-t)} \tau(s) \mathrm{d}s.$$

At the OQM, with $m = m_s$, the government must be giving negative transfers τ (that is, levying taxes) to make the *nominal* money supply shrink over time. (As I explained before, however, the individual considers this tax to be unrelated to his/her own level of money holdings.) The tax is equal to rm_s in real terms if real balances are constant at the satiation point m_s. Thus, the preceding budget constraint will hold if

$$m_s = -\int_t^\infty \mathrm{e}^{-r(s-t)} \left(-rm_s\right) \mathrm{d}s$$

(which in fact is a true equality). Basically, people are planning to use their real balances to pay off their future taxes.

Now we can see that the OQM equilibrium has a very bizarre property. Suppose we have $i = 0$ and the central bank unexpectedly deviates from its constant (negative) monetary growth rate It does so by a surprise gift of Δ

dollars to everyone. (These dollars are printed up overnight and stuffed in everyone's mailbox in the morning.) *In equilibrium, the price level P does not change, people's real balances rise from m_s to $m_s' = m_s + \frac{\Delta}{P}$, and the economy continues on as before, simply with a higher level of real balances (still yielding a marginal utility of zero).* To see this, note that the new criterion for the individual budget constraint to hold in equilibrium is simply that

$$m_s' = -\int_t^\infty \mathrm{e}^{-r(s-t)} \left(-rm_s'\right) \mathrm{d}s,$$

which of course is (still) true. People happily add to their real balances because, in the new equilibrium, expected future taxes are higher (as they have to be to bring about the same shrinkage *rate* of a higher money-supply *level*). Along with the new money that people find in their mailboxes, they also find a notice of higher future taxes. As a result, they do not spend the newly found money, which, at $i = 0$, is perfectly substitutable for bonds. That is why P does not rise.[5]

This equilibrium is one example (a flexible-price example) of the *liquidity trap*. At a zero nominal interest rate, a money-supply increase does not affect the economy. You are probably familiar with the Keynesian liquidity trap in the IS-LM model, but the basic idea is more general. One reason real-world policymakers target positive inflation rates (rather than following the OQM) is that they wish to remain far away from the zero-interest trap (a place Japan, for example, was in until recently). At that point, interest rates cannot be cut and open-market operations may lose their effects.

Dynamic inconsistency: Temptation and redemption

Once upon a time, inflation was high — quite high in most industrial economies, very high in much of the developing world. Starting around 1990, inflation rates worldwide began to come down, and although there are still

[5] In the debate over Japanese monetary policy under the zero nominal interest rate that prevailed for many years between 1995 and 2006, some economists suggested that, while open-market operations could not affect the economy, gifts of money to the public, financed by transfer payments, would raise private-sector wealth and thereby spending. The preceding analysis, however, shows the fallacy in this position.

some notable recent examples of hyperinflation (Zimbabwe comes to mind), very high inflation has become much rarer than it was in the 1970s and 1980s. Some of the credit, I believe, should go to the models I will discuss next.

Consider first the government's budgetary problem. If the government could carry out a one-time surprise inflation (for example, by unexpectedly buying back large block of government debt with newly printed money), society might be better off. The government could then lower distorting taxes (due to a lower outstanding debt). What the government has done is effectively to levy a surprise tax on people's pre-existing stock of real money balances. Because expectations of the future are not affected, nominal interest rates can remain the same, and no additional distortion on the demand for money is introduced,

The problem with this scenario is that if it pays for the government to do it once, it pays to do it again. And again. The government may promise never to inflict another surprise inflation, but the public knows that the temptation is too great. Even though it is in society's *collective* interest to allow the government to spring a surprise, it is in each individual's *personal* interest to protect himself or herself from inflation. There is a Prisoner's Dilemma. As a result, nominal interest rates will rise to high levels, swelling the government's expenses and inflicting welfare losses. A promise by the government to avoid inflation surprises is not credible; it would be *dynamically inconsistent.* This type of problem was first analyzed for a monetary economy in a famous November 1978 *Econometrica* paper by Guillermo Calvo.

A different type of dynamic inconsistency problem relates to the government's efforts to manage aggregate employment through monetary policy. Finn Kydland and Edward Prescott analyzed this scenario in a celebrated June 1977 paper in the *Journal of Political Economy.* (This paper was half the reason they shared the Nobel Prize in economics.) My discussion is, however, based on the exposition by Robert Barro and David Gordon (in the course reader, or *Journal of Political Economy*, August 1983).

Barro-Gordon model. The economy consists of two sets of agents the monetary authorities (or central bank) and wage setters.

The loss function of the authorities is

$$\mathcal{L} = (y - y^*)^2 + \beta\pi^2, \tag{1}$$

where y^* is the authorities' *target* level of output. They wish to minimize deviations of actual from target output, as well as to minimize deviations of the inflation rate from zero. We assume $\beta > 0$.

Output is given by the Phillips curve relation

$$y = \bar{y} + \alpha(\pi - \mathrm{E}\pi) + u, \tag{2}$$

where $\bar{y}$ is the "natural" level of output and u is a random mean-zero, i.i.d. shock. A critical assumption of the model is that

$$y^* > \bar{y}.$$

Perhaps because there are distortions in the economy (such as market power) that tend to depress output, the authorities target a level of output above the natural rate. In adopting this target, the authorities may be motivated by a desire to raise public welfare, but as in other examples of dynamic inconsistency, their good intentions can lead to bad outcomes.

The Phillips curve (2) arises as follows. A period in advance of market activity, workers set a nominal wage equal to their current wage plus expected inflation $\mathrm{E}\pi$. Inflation above expectations thus lowers the real wage, raising employment and output.

We can think of different "games" that might be played between the two "players" — the workers and the monetary authorities. One game is a "precommitment game" in which the authorities have the capacity to commit themselves to a specific *monetary policy rule* (in this case a "feedback" rule that depends on the realization of u). On the other hand, we can also imagine a Nash game where there is no commitment, so that the authorities' "move" (choice of π) can depend on how workers have previously "moved" (by setting nominal wages based on $\mathrm{E}\pi$).

Precommitment game. In this game the authorities can credibly promise to always follow a rule of the form

$$\pi = \mu_0 + \mu_1 u,$$

where μ_0 and μ_1 are constants. (Importantly, the authorities are committing in advance to a *formula* that will govern how they choose inflation in the future, and not to any specific level of the inflation rate for the future. We will find the optimal rule in a moment.) They choose this rule *before* u is realized and *before* workers form expectations. Under this setup, inflation does not depend on workers' expectations. Accordingly, the problem of the workers is easy — they set $\mathrm{E}\pi = \mu_0$.

Under the rule, therefore

$$y = \bar{y} + \alpha(\mu_0 + \mu_1 u - \mu_0) + u = \bar{y} + (1 + \alpha\mu_1)\, u$$

and the authorities' loss can be written as

$$\mathcal{L} = [\bar{y} + (1 + \alpha\mu_1)\, u - y^*]^2 + \beta\, (\mu_0 + \mu_1 u)^2 .$$

Given the timing of the authorities' choice, however, the best they can do is to choose the rule parameters μ_0 and μ_1 to minimize

$$\begin{aligned} \mathrm{E}\mathcal{L} &= \mathrm{E}\left\{[\bar{y} + (1 + \alpha\mu_1)\, u - y^*]^2 + \beta\, (\mu_0 + \mu_1 u)^2\right\} \\ &= (y^* - \bar{y})^2 + \beta\mu_0^2 + \left[\beta\mu_1^2 + (1 + \alpha\mu_1)^2\right] \sigma_u^2. \end{aligned}$$

It is obvious that minimization of this expected loss requires that $\mu_0 = 0$. That finding implies that $\mathrm{E}\pi = 0$ if the policy rule is chosen optimally. Optimal μ_1 is derived from the first-order condition

$$\frac{\mathrm{d}\mathrm{E}\mathcal{L}}{\mathrm{d}\mu_1} = 0 = 2\beta\mu_1\sigma_u^2 + 2\alpha\,(1 + \alpha\mu_1)\,\sigma_u^2.$$

The solution is

$$\mu_1 = -\frac{\alpha}{\alpha^2 + \beta}.$$

The monetary authority will react to a negative u by increasing inflation somewhat beyond expectations, but its response is tempered by its aversion β to inflation. Only if $\beta = 0$ does the monetary authority fully offset the effect of u on y by setting $\pi = -u/\alpha$. Under the optimal rule, expected loss is

$$\mathrm{E}\mathcal{L}^R = (y^* - \bar{y})^2 + \frac{\beta}{\alpha^2 + \beta}\sigma_u^2$$

and, as noted above, expected inflation is zero.

Nash game (discretionary equilibrium). Now the order of moves by the players is as follows:

1. Prior to the realization of the shock u, workers set the rate of nominal wage increase $\mathrm{E}\pi$. (That is, the expectation $\mathrm{E}\pi$ is not conditioned on the period's value of u.)

2. Given the choice of $\mathrm{E}\pi$ by the workers, and after observing the realization of u, the authorities choose π so as to minimize the loss function (1). (Thus, the choice of π can depend on $\mathrm{E}\pi$ as well as on u.)

The Nash equilibrium of this game has the following fixed point property: $\mathrm{E}\pi$ must be the rational expectation (not conditioned on u) of the π the authorities find it optimal to choose given u and $\mathrm{E}\pi$. That is, if it is optimal to choose $\pi = \Pi(u, \mathrm{E}\pi)$, then

$$\mathrm{E}\Pi(u, \mathrm{E}\pi) = \mathrm{E}\pi.$$

Let us solve the game by backward induction. Given u and $\mathrm{E}\pi$, the authorities solve

$$\min_{\pi} \mathcal{L} = [\bar{y} + \alpha(\pi - \mathrm{E}\pi) + u - y^*]^2 + \beta\pi^2.$$

The first-order condition for a minimum is

$$\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}\pi} = 0 = 2\alpha\,[\bar{y} + \alpha(\pi - \mathrm{E}\pi) + u - y^*] + 2\beta\pi,$$

or

$$\pi = \frac{\alpha}{\alpha^2 + \beta}\,(y^* - \bar{y} + \alpha\mathrm{E}\pi - u) = \Pi(u, \mathrm{E}\pi). \tag{3}$$

In comparison to the optimal rule (with commitment), which was

$$\pi = -\frac{\alpha}{\alpha^2 + \beta}u,$$

under the present assumption of *discretion* in monetary policy, the authorities both choose higher inflation when expected inflation is higher (to avoid the resulting high real wages and low output) and attempt to move output above its natural rate $\bar{y}$ closer to the target level y^*. Their zeal to attain higher output is tempered by the cost of inflation (captured by parameter β in the loss function).

Now we take a step backward in time and ask what the workers would rationally expect, given the authorities reaction function in (3). The Nash equilibrium expected inflation rate is defined by the fixed point of

$$\mathrm{E}\pi = \mathrm{E}\left[\frac{\alpha}{\alpha^2+\beta}\left(y^* - \bar{y} + \alpha\mathrm{E}\pi - u\right)\right] = \frac{\alpha}{\alpha^2+\beta}\left(y^* - \bar{y} + \alpha\mathrm{E}\pi\right),$$

or

$$\mathrm{E}\pi = \frac{\alpha}{\beta}\left(y^* - \bar{y}\right).$$

Notice that if there is no concern for inflation ($\beta = 0$), equilibrium expected inflation is infinite. Nothing will deter the authorities from trying to raise output above the natural rate, and wages and inflation will chase each other upward in an unbounded spiral. If $\beta > 0$, however, there comes a point of finite expected inflation at which *equilibrium* inflation is so high that the monetary authorities have no further incentive to raise it.

Equilibrium inflation can be found by substituting $\mathrm{E}\pi = (\alpha/\beta)\left(y^* - \bar{y}\right)$ into equation (3). The result (naturally) is

$$\pi = \frac{\alpha}{\beta}\left(y^* - \bar{y}\right) - \frac{\alpha}{\alpha^2+\beta}u.$$

The authorities respond to shocks as they would under an optimal rule, but they also are caught in an inflationary trap by their own proclivity to create surprise inflation — which of course they cannot systematically do in a rational-expectations equilibrium. Intuitively, the expected loss under discretion (i.e., in the Nash game) is higher than under precommitment. It is

$$\mathrm{E}\mathcal{L}^D = \mathrm{E}\mathcal{L}^R + \frac{\alpha^2}{\beta}\left(y^* - \bar{y}\right)^2.$$

Possible ways to mitigate this dynamic inconsistency problem. Several remedies have been proposed. At some level, schemes of central bank independence and formal inflation targeting contain elements of these.

1. Conservative central banker (Kenneth Rogoff, *Quarterly Journal of Economics*, November 1985). Pick a man or woman to run the central bank who personally has an aversion to inflation measured by a β' that exceeds society's inflation aversion, β. Give the central banker independence to choose π. This can actually help *society* to attain a lower expected loss in equilibrium, even though its preferences depend on β rather than the higher β'. An issue is whether the political powers-that-be might somehow pressure or bribe the conservative central banker. Picture Alan Greenspan sitting with Hillary at Bill's state of the union address; and why *did* the maestro endorse the Bush tax cuts, anyway?

2. Central banker incentive contract (Carl Walsh, *American Economic Review*, March 1995). Give the central banker a compensation contract that penalizes him/her financially if inflation is too high. Problem: the political powers have to commit to pay neither more nor less than the contract amount, so the assumption of precommitment slips in through the back door.

3. Reputation (Barro and Gordon, *Journal of Monetary Economics*, July 1983). If higher than optimal inflation is "punished" by a period of very high expected inflation, the monetary authority may be deterred from deviating too much from an optimal rule, even if allowed discretion. The problem with this approach is that it relies on the possibility of multiple equilibria. Because the level of punishment is rather arbirtrary, the possible equilibria involve a range of inflation rates, some low, but some not far from the one that comes out of the simplest Nash equilibrium (without punishments) that we examined above. Of course, this is a consequence of the "folk theorem" of repeated games. In contrast, the two other fixes listed previously are based on institutional reforms and therefore may function more reliably to lower equilibrium inflation.

Notwithstanding the reservations I have expressed, there has been great progress in taming inflation. The decline in worldwide inflation since the early 1990s must in part be credited, I believe, to a deeper and broader understanding of the dynamic inconsistency problem, and to institutional reforms (including widespread central-bank independence from the rest of the government) that have evolved since then. Of course, these institutional reforms have not always worked perfectly — there is still a process of refinement and learning, and open questions remain. To take a currently relevant example, what should be the role of central banks in overseeing the stability of financial systems and in providing liquidity assistance to banks and asset markets in times of stress? Nonetheless, any reading of today's financial press reveals a level of economic literacy on inflation problems that far exceeds the level prevailing in the mid-1970s, when the seminal dynamic inconsistency papers of Calvo and Kydland-Prescott were written.