Lecture V **Economics 202A** **Fall 2007**

Section sign-ups at end of class.

Logs: for those of you who feel overworked (which is probably almost everybody) you may submit *two*, rather than *three,* logs over the next three weeks.

We have now finished Lucas and Sargent.

September 11: Taylor model
September 13: 3 questionnaire articles: Kahneman *et al,* Shafir *et al*, Shiller
September 18: Yellen and Shapiro and Stiglitz on Efficiency Wages
September 20: no class
September 25: Akerlof and Yellen on Near Rationality.
September 27: Fehr and Tyran on experimental general equilibrium & Mankiw on menu costs.
October 2: Miller & Orr on Demand for Money
October 4: Akerlof on Irving Fisher on Head; Caballero, Engel & Haltiwanger on demand for "durables."
October 9: Dornbusch on exchange rates

This takes us to the next starred item on the reading list.

I want to start where we ended last time, a bit more hurriedly than expected.

I am going to give a very quick summary.

There are two ingredients for Sargent's result:

First there are rational expectations.

Because of these rational expectations the expected price level at time t, which Sargent calls ${}_tp^*_{t-1}$, mimics p_t except for a white-noise error term.

As a result the gap between actual inflation and expected inflation is just a white noise error term, ϵ_t.

But then there is another assumption, which is the absence of money illusion.

Because there is no money illusion, the Phillips curve depends *critically* on the difference between actual and expected inflation, plus another error term.

This is equivalent to saying that the deviation from full employment depends on the gap between actual and expected inflation, plus this extra error term.

As a result we find, robustly, no matter what we do, so long as we have such an aggregate supply equation, that the gap between actual and full employment is of the form:

$$\gamma\epsilon_t + u_t.$$

So that is Sargent's result.

Looking at this formula tells us that there is no serial correlation of output. And there is no systematic effect of monetary policy.

But there is a critical assumption here.

In reality, aggregate supply *may* depend on the *price level* because there is some form of money illusion such as *sticky money wages.*

If you believe that money wages may be sticky in some form or other, then you have probably rejected Sargent's model and his conclusion of the neutrality of the monetary rule.

The key assumption in his model then is the absence of money illusion.

That is the beginning of today's lecture.

The most standard answer to rational expectations is the model by John Taylor.

Taylor's model is a model of *rational expectations*, but it also has *money illusion* of a natural sort.

Lucas and Sargent emphasize *rational expectations*.

But in fact their strongest assumption is probably the total *absence* of money illusion.

That absence is the most fundamental reason for the neutrality of money and the lack of serial correlation of output in their model.

In contrast, in Taylor's model there is *serial correlation of output* and also *monetary policy* that is *effective in stabilizing output.*

We shall find three things:

1. With *RE* and with a *little bit* of money illusion, as in the Taylor model, monetary policy will be effective.

2. There will also be serial correlation of output (even in the absence of serially correlated supply shocks).

3. Also since, the economy is linear, as in Lucas' and Sargent's equations, it will be easy to solve.

The simple model by Taylor illustrates all three of these points.

I shall describe Taylor's model:

His model has *unsynchronized* price setting.

> **½ of the firms set their prices in *even* periods.**
> **½ of the firms set their prices in *odd* periods.**

Measuring time in 6-month periods we might view

> **½ of the firms as setting their price every *January,***

and ½ of the firms as setting their prices every *July.*

Money illusion is introduced in the following way:

> ***nominal* prices are *constant* over the two-period interval.**

A firm setting a price for time t, sets its price *both* for time t and for time t+1.
Its pricing decision is based on information available at t-1.

How could a two-period contract be different?
The contract made at t *could* specify prices at t+1 *contingent* on information *that only becomes available at t+1.*

For example: labor contracts may be *indexed* by the *cost of living.*
In that case the wages in the second period of the contract are contingent on information available only after the contract is made.

We are assuming that does *not* happen here.
Good studies have been made of Canadian union contracts.
Curiously, *most* of these contracts are *not* indexed.
Even then they are not fully indexed.

Let me continue describing Taylor's model.

I am going to give you the equations of the model and their justification.

The microeconomic basis for the model assumes that there are just two firms. The demand for the product of each firm depends on its own price, on the price of its competitor's product, and on aggregate demand.

To be simple, assume that these firms have no costs of production—so the firms setting prices will try to maximize *expected discounted revenues.*

USE RHBB TO LIST VARIABLES
WRITE SYMBOLS AS YOU GO – WITH SPACES

Now let's adopt a clever notation.
Let x_t be the price which is set at time t.
We will consider one of these firms that is setting its price at time t.
The firm setting its price at time t will see that its demand in the current period will depend on the price of its competitor, or x_{t-1}.
We have a t-1 subscript because *that* price was set last period, at t-1.
The *expected demand* for the firm setting its price at t will also depend on *expected aggregate demand.* We shall denote this as $\hat{y}_t$.
ˆ in Taylor's notation denotes *expectations made on the basis of t-1 information.*

The firm's expected demand at t+1 will also depend on the expected value of the competitor's price at t+1, which will be the expected price set next period or

$\hat{x}_{t+1}$.

The firm's expected demand will also depend on the expected value of aggregate demand next period, which will be

$\hat{y}_{t+1}$.

Let's not be worried about the functional form.
The firm that sets the nominal price over the two periods to maximize profits will then set:

<FAR RHBB; divide in ½ by line>

$$(1) \qquad x_t = b\, x_{t-1} + d\, \hat{y}_t + \gamma\, (b\, \hat{x}_{t+1} + d\, \hat{y}_{t+1}) + \epsilon_t$$

where ϵ_t is an additional random variable term in the firm's prices due to the *mistakes* in its pricing behavior.

The notation is in logarithms:

> **x_t is the log of the price set at time t**
> **x_{t-1} is the log of the price set by the competitor at t-1**

$\hat{y}_t$ is the log of expected income at t
$\hat{x}_{t+1}$ is the log of the expected price set by the competitor at t+1
$\hat{y}_{t+1}$ is the log of expected income at t+1.

This is Taylor's key equation.

FOOTNOTE ON NOTATION: d is a constant coefficient. It does not represent a differential or anything fancy. END NOTE

ERASE LHBB

Now let's add some standard stuff – keeping things very simple to complete the model.

By definition the aggregate price level will be

(2) $w_t = \frac{1}{2} x_t + \frac{1}{2} x_{t-1}$

That is, the aggregate price level is a weighted average of prices set *this* period and prices set *last* period.

To be precise: w_t is the log of the aggregate price level.
So we are taking a geometric mean rather than an arithmetic mean.

And suppose further that aggregate demand is determined by a *Quantity Theoretic* equation:

(3) $y_t = m_t - w_t + v_t$

where m_t is the log of money balances
w_t is the log of the aggregate price level
v_t is the random error term.

Now let's introduce a money supply rule.
The monetary authority is *stabilizing.*

This means that as the price level w_t goes up, Real Balances, which determine aggregate demand, should go down.

As a result the monetary authority has the rule:

(4) $m_t = (1 - \beta) w_t$ $0 < \beta < 1$.

We now have the *whole model.*

There are two *salient questions.*
These questions are motivated by Lucas and Sargent.

(1) Is this monetary policy *stabilizing*, with *rational expectations*?

(2) Is there serial correlation in *output* and *prices* so that there is a business cycle?

Does this occur even in the absence of serially correlated supply shocks?

Since these equations may look a bit unfamiliar to you, it is useful to do a short review so that you can see why they describe the whole economy.

Equation (1) is an aggregate supply equation.
It gives an *admittedly complicated* relation between income and price.
It tells what price firms will charge given expected income $\hat{y}_t$ and $\hat{y}_{t+1}$.

Equation (3) is an aggregate *demand* curve.
Given the money supply, it tells the relation between price and aggregate goods demanded. Aggregate goods demanded will be aggregate income.

So equations (1) and (3) are the Aggregate Demand and Aggregate Supply curves, given the supply of money.

If you know *money,* you can determine *prices* and *aggregate demand.*

Then equation (4) tells you how much *money* there is.

Equation (2) links the definition of prices in (1) to the definition of prices in (3).

David Romer has a particularly clever way of solving the model, which is at the end of this lecture.
Let me now solve the model as Taylor does.

Let's do a little algebra and see if we can solve the model.
We will first solve the model in detail.

We will first take the bird's eye view.
Then we will come back and describe in general terms what we have done, with a broader perspective.

To solve such a model we would want to express all the endogenous variables in

terms of current and past shocks.

What are those shocks. They are the ϵ_t's (and possibly also the v_t's, as well).

If we can get formulas in terms of the random shocks, then we know everything there is to know about the model.

Before we solve the model let's think about what we mean by a solution.

We know the basic equation of the model:

$$x_t = b\,x_{t-1} + d\,\hat{y}_t + \gamma\,(b\,\hat{x}_{t+1} + d\,\hat{y}_{t+1}) + \epsilon_t$$

If we knew $\hat{y}_t$, $\hat{x}_{t+1}$, and $\hat{y}_{t+1}$ we could plug them in, and then we would know the actual value of x_t by solving the difference equation that would result.

If you think about it for a while you will realize that you can find the values of $\hat{y}_t$, $\hat{x}_{t+1}$, and $\hat{y}_{t+1}$ that are consistent with the structure of the economy.

It turns out that people knowing the structure of the economy will have expectations about

$$\hat{y}_t,\ \hat{x}_{t+1},\ \text{and}\ \hat{y}_{t+1}\ .$$

It turns out that these *consistent* or *rational expectations* must be solutions to a difference equation, which we will now set up and *solve.*

<DIVIDE LHBB IN 2;
WARNING: NEED SEVEN LINES ON ONE PANEL.>

We know from the quantity theory equation:

$$y_t = m_t - w_t + v_t \qquad \text{So} \qquad \hat{y}_t = \hat{m}_t - \hat{w}_t\ .$$

And we know the monetary rule:

$$m_t = (1 - \beta)\,w_t \qquad \text{So} \qquad \hat{m}_t = (1 - \beta)\,\hat{w}_t$$

$$\text{So} \qquad \hat{y}_t = (1 - \beta)\,\hat{w}_t - \hat{w}_t = -\beta\,\hat{w}_t$$

$$\text{And similarly} \qquad \hat{y}_{t+1} = -\beta\,\hat{w}_{t+1}$$

NEXT PANEL ON RHBB

We also know

$w_t = .5\, x_{t-1} + .5\, x_t$ So $\hat{w}_t = .5\, x_{t-1} + .5\, \hat{x}_t$

GO TO LHS OF LHBB

So $\hat{y}_t = -\beta\,(.5\, x_{t-1} + .5\, \hat{x}_t)$

And $\hat{y}_{t+1} = -\beta\,(.5\, \hat{x}_t + .5\, \hat{x}_{t+1})$

So if we want to discover $\hat{x}_t$, $\hat{y}_t$, $\hat{y}_{t+1}$ and $\hat{x}_{t+1}$ all we need to know is the path of the $\hat{x}_t$'s.

Go over to RHBB

Since $x_t = b\, x_{t-1} + d\, \hat{y}_t + \gamma\,(b\, \hat{x}_{t+1} + d\, \hat{y}_{t+1}) + \epsilon_t$

ERASE RHBB below this equation

and since ϵ_t is not known for all times s > t-1, it follows that for s > t-1

$$\hat{x}_s = b\, \hat{x}_{s-1} + d\, \hat{y}_s + \gamma\,(b\, \hat{x}_{s+1} + d\, \hat{y}_{s+1}).$$

The reason: according to rational expectations the behavior of expectations must conform to the behavior of the system.

Formally you obtain this difference equation by taking the $E|\theta_{t-1}$ of the LHS and of the RHS of the equation for s>t-1.

Substituting the value of $\hat{y}_s$ **in terms of** $\hat{x}_s$ **and** $\hat{x}_{s+1}$
AND $\hat{y}_{s+1}$ **in terms of** $\hat{x}_{s+1}$ **and** $\hat{x}_{s+2}$

POINT TO TWO EQUATIONS ON LHBB; ERASE LHBB

yields a difference equation in terms of $\hat{x}_s$.

That difference equation is of the form:

$$c\, \hat{x}_s = \hat{x}_{s-1} + \gamma\, \hat{x}_{s+1}$$

where c is a complicated constant, which comes out of the algebra.

FOOTNOTE:

END FOOTNOTE.

This constant is obtained just by substituting into the equation for $\hat{x}_s$ the respective formulae for all the variables on the RHS and then gathering terms.

NOW ERASE LHBB leaving only the difference equation.

This difference equation yields the *expected path of prices consistent with the model.*

It is a second order difference equation with solution of the form:

$$\hat{x}_s = a_1\, \alpha_1^{\,s} + a_2\, \alpha_2^{\,s},$$

where

and

α_1 and α_2 are the roots of the associated quadratic of the difference equation.

We know this from our Difference Equation BOX for the solution to the second order linear difference equation.

That associated quadratic WRITE IT UNDER THE EQUATION is:

$$\gamma x^2 - cx + 1 = 0$$

The larger of the two roots, which is α_2 has absolute value greater than one.

FOOTNOTE:

Proof: $\alpha_1 \cdot \alpha_2 = 1/\gamma > 1$. END FOOTNOTE

In consequence, if the economy is well-behaved in the long-run a_2 must be zero.

Why? In the long-run this root will dominate if a_2 is not zero.

If a_2 is not zero, the log of income and the log of prices will behave very erratically in the long run. We impose the additional condition that we want a long-run solution *for these expectations* that, qualitatively, makes good economic sense.

If we impose the assumption of long-run stability on the system, then $a_2 = 0$ and

$$\hat{x}_s = a_1 \alpha_1^s.$$

And at time t-1 we have the initial condition

$$\hat{x}_{t-1} = x_{t-1}.$$

So $$\hat{x}_s = \alpha_1^{s-(t-1)} x_{t-1}.$$

In particular,

$$\hat{x}_{t-1} = x_{t-1}$$

$$\hat{x}_t = \alpha_1 x_{t-1}$$

$$\hat{x}_{t+1} = \alpha_1^2 x_{t-1}$$

If we substitute these values, and also the associated values for

$\hat{y}_t$ and $\hat{y}_{t+1}$ into the fundamental pricing equation we find

the elegant solution

$$x_t = \alpha_1 x_{t-1} + \epsilon_t .$$

This leads immediately to *the solution* to the model.

We can solve this difference equation, as you may remember, and get an equation for x_t in terms of current and past shocks. The solution comes from our very first difference equation: the first one we put into a BOX.

And with a bit of algebra you can compute any other variable in the model once you know the value of the x_t's.

Especially, you can compute the y_t's.
You can show easily that they are serially correlated.
You can also show that they depend upon monetary policy, because the value of y_t will depend upon the value of β.
Remember that β is the parameter that describes monetary policy.

This completes the solution to the model.

ERASE LHBB EXCEPT FOR (1)
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Having completed, in detail, the solution to the model, let me now go back and review what we did, in a nutshell.

And then I also want to discuss at greater length why we chose the *smaller* root to the difference equation.

Let me *review* what we did–so you can see the broader picture.

We started with the basic pricing equation:

(1) $$x_t = bx_{t-1} + d\,\hat{y}_t + \gamma\,(b\,\hat{x}_{t+1} + d\,\hat{y}_{t+1}) + \epsilon_t$$

By finding out how $\hat{y}_t$ depended on $\hat{x}_{t-1}$ and $\hat{x}_t$

and $\hat{y}_{t+1}$ depended on $\hat{x}_t$ and $\hat{x}_{t+1}$

and substituting in this equation, we get a difference equation consistent with the pricing behavior (1).
This difference equation determined what *consistent price* expectations would be.

This difference equation had a solution of the form:

$$\hat{x}_s = a_1\,\alpha_1^{\,s} + a_2\,\alpha_2^{\,s}.$$

By definition, let's assume that α_2 is the larger root and that it is greater than one.

Then a_2 the coefficient associated with the larger root must be zero. It must be zero because otherwise as t grows large the system behaves very erratically.
Let me go over that argument in greater detail than I have so far.

Why do we throw out the larger root?
Let's go back and look at the system.
The system here has a steady state equilibrium if the stochastic terms are 0.
We can find this steady state.
This steady state will have constant money, income, and prices.

We assume that people will have rational expectations and assume that the system will approach this steady state in the absence of exogenous shocks.

Why not allow weight on the root that is greater than one?

In that case in the long run the solution is not stable. In fact the variance of income growth becomes *larger and larger.*

$$\sigma^2 (y_t - y_{t-1}) \rightarrow \infty \text{ as } t \rightarrow \infty.$$

For this reason solutions to the difference equation with nonzero weight on α_2 should be rejected.

I will give the proof as a footnote in the lectures.

FOOTNOTE:

Proof. $y_t = m_t - w_t + v_t$

$= (1 - \beta) w_t - w_t + v_t$

$= -\beta w_t + v_t.$

Similarly $y_{t-1} = -\beta w_{t-1} + v_{t-1}.$

$$E(y_t \cdot y_{t-1}) = \beta^2 E(w_t \cdot w_{t-1})$$

But $w_t = .5(x_t + x_{t-1})$

Asymptotically,

$$\cong .5(\alpha_2^2 x_{t-2} + \alpha_2 \epsilon_{t-1} + \epsilon_t + \alpha_2 x_{t-2} + \epsilon_{t-1})$$

$$= \alpha_2 .5(\alpha_2 x_{t-2} + \epsilon_{t-1} + x_{t-2}) + .5(\epsilon_t + \epsilon_{t-1})$$

$$= \alpha_2 .5(x_{t-1} + x_{t-2}) + .5(\epsilon_t + \epsilon_{t-1}).$$

We can therefore view as an approximation, if $a_2 \neq 0$

$$w_t = \alpha_2 w_{t-1} + .5(\epsilon_t + \epsilon_{t-1})$$

The growth in income is then:

$$y_t - y_{t-1} = -\beta w_t + v_t + \beta w_{t-1} - v_{t-1}$$

$$= -\beta(w_t - w_{t-1}) + v_t - v_{t-1}$$

$$\sigma^2 (y_t - y_{t-1}) = \beta^2 \sigma^2 (w_t - w_{t-1}) + 2\sigma_v^2$$

$$w_t - w_{t-1} = (\alpha_2 - 1)\, w_{t-1} + .5(\epsilon_t + \epsilon_{t-1})$$

$$\sigma^2 (w_t - w_{t-1}) = (\alpha_2 - 1)^2\ \sigma^2 (w_{t-1}) + (\alpha_2 - 1)\,(.5)\ \mathrm{cov}\,[(\epsilon_t + \epsilon_{t-1}), w_{t-1}] + .5\sigma_\epsilon^2.$$

$\sigma^2 (w_{t-1})$, however, is ever increasing in time because

$$w_t = \Sigma_{i=1}^{t}\, \alpha_2^{\,t-i} (\epsilon_i + \epsilon_{i-1}) + \alpha_2^{\,t}\, w_0$$

By inspection $\sigma^2 (w_t)$ will be ever increasing if $\alpha_2 > 0$.

END FOOTNOTE

So the growth in income will have ever increasing variance as t increases. If we reject this instability, then a_2 must be zero.

It is highly peculiar for an economy with no technical change to have long-run growth in income which is either extremely large or extremely small. This will occur if

$\sigma^2 (y_t - y_{t-1})$ is unbounded.

So let's return to our review of how we got our solution.
The solution is of the form:

$$\hat{x}_s = a_1\, \alpha_1^{\,s}.$$

With the initial condition

$$\hat{x}_{t-1} = x_{t-1}$$

we find that

$$\hat{x}_s = \alpha_1^{\,s-(t-1)}\, x_{t-1} \quad \text{for } s>t-1.$$

We then know

$$\hat{x}_{t-1} = x_{t-1}$$

$$\hat{x}_t = \alpha_1\, x_{t-1}$$

$$\hat{x}_{t+1} = \alpha_1^{\,2}\, x_{t-1}$$

Solving for the implicit values of $\hat{y}_t$ and $\hat{y}_{t+1}$ and substituting for $\hat{x}_{t-1}$ and $\hat{x}_{t+1}$ into (1) yields

$$x_t = \alpha_1 x_{t-1} + \epsilon_t.$$

This is absolutely wonderful because this equation is an *old friend.*

We know everything about her. She is just the AR(1) equation.

***First,* either because we proved it in class or because we can easily work it out, we know its solution:**

$$x_t = \Sigma_{i=0}^{\infty} \alpha_1^{\ i} \epsilon_{t-i.}$$

And, *second,* we know that there will be *serial correlation* because we have already worked out the autocorrelation function of this AR(1) process.

***Third,* we can write out what y_t will be in an explicit formula, and we can compute the covariance between y_t and y_{t-1}.**

You find that y_t is serially correlated.

***Fourth,* you can calculate the variance of y_t as a function of the monetary rule described by the parameter β.**

You will find that the variance of y_t changes with β and therefore systematic monetary policy in this model may be stabilizing.

How do you know this? Because the calculated variance y_t depends on α_1. And in turn α_1 depends on the parameter β, which describes monetary policy.

What is the message of this model? The message of this model is that *Rational expectations models with money illusion,* (but without serially correlated supply shocks), may have *serially correlated output.*

The further message is that *monetary policy* may be *stabilizing* in such an economy.

Sargent and Lucas would not like this model: because the agents are not maximizing.
It is not maximizing for the price to be set the same in nominal terms for two periods.

If price setters were maximizing, the *price* in the second period would be

contingent **on information that becomes newly available.**

This is a Keynesian model, in the sense that nominal prices are a bit sticky. This small amount of stickiness in prices makes monetary policy *effective* in stabilizing output.
It also makes output serially correlated.

BEGIN SKIP (unless there is time)

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I will now skip in class (but not in the lecture notes) my review of David Romer's excellent coverage of this model, which is very different.

He also has a very clever way to solve it.

David has an alternative version of the Taylor model, and also an alternative way to solve it.

I am going to go over that to show you a different method of solution: matching coefficients.

First, for simplicity, his monetary rule is that money follows a random walk:

$m_t = m_{t-1} + \epsilon_t$.

From microeconomic assumptions he derives the optimal price charged by the individual firm.
This is the price the individual firm would ideally like to charge for its product at time t.

$p^*_{it} = \phi\, m_t + (1 - \phi)\, p_t,$

where p_t is the aggregate price level at t.

Note that the equation is normalized so that if m_t were constant p_t would be equal to m_t.
This is equivalent to treating p_t and m_t as index numbers equal to 100 in the base year.
Thus if m_t should increase by 5 percent to 105 we would expect p_t to increase to 105 as well.

This formula makes intuitive sense.
There are two factors that determine the price a firm wants to set.

One is demand, which is determined by the money supply.
The other is the price set by competitors, which is on average, p_t.

Returning to the pricing equation, we see further that if the money supply should double and the price set by the competition should double, the individual firm would want to double its own price.

So the price that the firm sets should be homogeneous of degree one in the money supply and the average price level.
Thus the coefficient on p_t should be one minus the coefficient on m_t.

Let's continue the analysis.

In the Taylor model the firm sets a constant price for two periods.
If the firm sets the price as close to the optimum as possible it will set:

$$x_t = \tfrac{1}{2}\left(p^*_{it} + E_t\, p^*_{it+1}\right)$$

As a result (using the fact that $p_t = x_{t-1} + x_t$)

$$\begin{aligned} x_t = \; & \tfrac{1}{2}\left[\phi\, m_t + (1-\phi)\, \tfrac{1}{2}\left(x_{t-1} + x_t\right)\right] \\ & + \tfrac{1}{2}\left[\phi\, E_t\, m_{t+1} + (1-\phi)\, \tfrac{1}{2}\, E_t\left(x_t + x_{t+1}\right)\right] \\ = \; & \tfrac{1}{2}\left[\phi\, m_t + (1-\phi)\, \tfrac{1}{2}\left(x_{t-1} + x_t\right)\right] \\ & + \tfrac{1}{2}\left[\phi\, m_t + (1-\phi)\, \tfrac{1}{2}\left(x_t + E_t\, x_{t+1}\right)\right] \\ = \; & \phi m_t + \tfrac{1}{4}\left(1-\phi\right)\left(x_{t-1} + 2x_t + E_t\, x_{t+1}\right) \end{aligned}$$

Note: in Romer's model x_t is known at t.

Solving for x_t yields

$$x_t = A\left(x_{t-1} + E_t\, x_{t+1}\right) + (1 - 2A)\, m_t$$

where

The algebra is straightforward.

Now let's use the method of matching coefficients to solve this equation.

Let's *guess* that the solution of the difference equation is of the form.

$$x_t = \lambda x_{t-1} + v m_t.$$

Now let's try our solution by the method of matching coefficients.
We will choose the values of those coefficients that will make our expectations "correct" or "rational" given the behavior of the model.

We know

$$x_t = \lambda x_{t-1} + v m_t$$

so $$E_t x_{t+1} = E_t (\lambda x_t + v m_{t+1})$$

$$= E_t \lambda(\lambda x_{t-1} + vm_t) + vm_t$$

$$= \lambda^2 x_{t-1} + (\lambda + 1) vm_t$$

Now remember

$$x_t = A (x_{t-1} + E_t x_{t+1}) + (1 - 2A) m_t$$

$$= A x_{t-1} + A \lambda^2 x_{t-1} + A(\lambda + 1) vm_t + (1 - 2A) m_t$$

Now remember our best guess:

$$x_t = \lambda x_{t-1} + v m_t.$$

So the coefficients must match:

$$\lambda = A + A \lambda^2$$

$$v = A (\lambda + 1) v + (1 - 2A)$$

And λ must solve the quadratic equation

$$\lambda = A + A \lambda^2.$$

Knowing λ we can use our second equation [POINT TO IT] to solve for the value of v.

[You can also check David's proposition that if $v = 1 - \lambda$, then the equation for v is satisfied. So $v = 1 - \lambda$.]

This solves the problem of the motion of prices.

As in Taylor, we find two solutions: two values of λ.
But we throw away the value of λ greater than one–since that is the unstable root.

We get the same type of results that we got before.

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Let me review the method and how it works.

You have two expressions for x_t.

One is by assumption that it follows a linear difference equation.

The other is that it follows the equation of motion of the system with $E_t\, x_{t+1}$ consistent with the linear difference equation.

The two equations must be consistent and this determines the coefficients of the linear difference equation.

The essence of that method was to do the *following three steps*.

1. Assume a *form* of the key variable.

Such as:

$$x_t = \lambda\, x_{t-1} + \nu\, m_t$$

2. We took the following from David Romer:

The price that firms ideally want to charge is:

$$p_{it}^* = \phi\, m_t + (1 - \phi)\, p_t,$$

where p_t is the aggregate price level at t.

The firm then setting its price at time t, x_t, will set:

$$x_t = ½\,(p^*_{it} + E_t\, p^*_{it+1})$$

As a result (using the fact that $p_t = x_{t-1} + x_t$)

We can find easily that:

x_t = A (x_{t-1} + E_t x_{t+1}) + (1 - 2A) m $_t$

where

The algebra is straightforward.

As a further step, if

x_t = λ x_{t-1} + ν m $_t$

with a bit of further algebra we can express E_t x_{t+1} in terms of λ and ν {point]. And since m_t is a random walk, we easily get a formula for x_t in terms of x_{t-1} and m_t.

That formula turns out to be:

x_t = A x_{t-1} + A λ^2 x_{t-1} + A(λ + 1) ν m $_t$ + (1 - 2A) m $_t$.

3. Then match the coefficients of

x_{t-1} with λ
and of m_t with ν.

This means that

λ = A + A λ^2

ν = A (λ + 1) ν + (1 - 2A)

So λ must solve the quadratic equation

λ = A + A λ^2.

Knowing λ we can use our second equation [POINT TO IT] to solve for the value of ν.

[You can also check David's proposition that if ν = 1 - λ, then the equation for ν is

satisfied. So $\nu = 1 - \lambda$.]

This solves the problem of the motion of prices.

As in Taylor, we find two solutions: two values of λ. But we throw away the value of λ greater than one–since that is the unstable root.

We get the same type of results that we got before.

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In a nutshell:

You have two expressions for x_t.

One is by assumption that it follows a linear difference equation.

The other is that it follows the equation of motion of the system with $E_t\, x_{t+1}$ consistent with the linear difference equation.

The two equations must be consistent and this determines the coefficients of the linear difference equation.

Note that this is one more technique for solving a model by using recursion.

A lot of this course is about using recursion.

****

END SKIP

I now have 10 REMARKS on Taylor.

I will probably have only time for four or five of these remarks in class. I will leave the remaining remarks in the notes.

Remark 1. **I want you to note that we were able to solve the model *totally* with rational expectations because the basic structure was *linear*.**

Remark 2. **You can think of this as a *rational expectations* version of a Keynesian model.**

It is *Keynesian* because prices are slow to change. It is a rational expectations model because decision makers have rational expectations.

In fact what I called *prices* Taylor calls *wages.* So this is a model in which

wages are slow to change.

You get the same model if you assume the following.

There are two *unions* in the economy. They work for two different firms (or two different industries).

One of these unions sets its *nominal* wage in even periods.
The other sets its *nominal* wage in odd periods.

The firms that hire these unionized workers face downward sloping demand curves.

The firms set the prices of their products as a mark-up then, as a mark-up over their marginal costs.
Those marginal costs are determined by the wages they have to pay their workers.

You get the same model then if you do that—just with a bit more unnecessary algebra.

Remark 3. Let me give you a bit of intuition about this system.

Suppose there were no stochasticness.
Up to time t_0 money had been *constant.* Then at time t_0 money jumps upward unexpectedly.

Let me give you a picture.

money

time

Let's put M on one axis and time on the other.
Draw axes.

Money is constant up to t_0.

Put in t_0.

Draw money constant up to t_0.

Money jumps unexpectedly at t_0 and is constant thereafter.

In the Lucas-Sargent model prices will adjust immediately.

In the Taylor model prices will be slow to adjust.
Let me explain why prices are *slow* to adjust.

Suppose I am the manager at one of the two firms in the Taylor model— this is one of the two firms that alternate in setting its respective price.

When I set *my* price I consider *two* nominal variables that are at least temporarily fixed.
***One* is the money supply.**
The *other* is the price that my competitor fixed in the last period.

Because I must consider her price, which is fixed, I will be slow to adjust in response to the change in the money supply.

***Then* in the next period, when my competitor sets *her* price, she will see that *my* price is fixed, so that she will be slow to adjust.**

For each price setter there are *two* nominal variables that are fixed: the competitor's price this period and money.

As a result prices* will adjust slowly to a *known money supply change.

Remark 4.* We have reviewed two important models with *Rational Expectations.

Three interesting questions have emerged in these models.

(1) Should a variable be unpredictable from previous information?

(2) Why might labor markets fail to clear?

(3) How does one solve systems with perfect foresight, or with rational expectations?

<Questionnaire regarding section times>.

***PROBABLY END REMARKS GIVEN IN CLASS* HERE**

***Remark 5.* There is something peculiar in Taylor's version of his model although not in David Romer's version.**

The key equation at time t is:

(1) $$x_t = bx_{t-1} + d\,\hat{y}_t + \gamma\,(b\,\hat{x}_{t+1} + d\,\hat{y}_{t+1}) + \epsilon_t.$$

The ϵ_t error terms are responsible for
(1) the serial correlation in income
and (2) the relation between β and the stabilization of output.

If $\sigma_\epsilon^2 = 0$ then there is no output stabilization from the monetary rule, and no serial correlation in income.

In my view the parable told in this model is that there are many industries. And *each industry* sets its price according to the key equation (1). The competition for each monopolistic competitor is *all the other goods* in the economy.

For example, the competition for the housing industry includes the skiing industry, the Saran wrap industry and the industry for cat food. Each of these industries is a monopolistic competitor vying for the consumer dollar.

This yields *staggered* price setting, as pictured for the following reason: Suppose that you as a monopolistic competitor set your price *once a year*.

If your competitor's timing of price change is uniform over the course of the year, then *on average* they will have set their prices *six months ago.*

Thus it makes sense to view each individual price setter as setting *her* price once a year. And it also makes sense to view her competitors as having set their prices six months ago.

Let me now explain why this interpretation causes problems for Taylor's model. Because, if there are *many* industries setting their prices, the ϵ error terms which are presumably independent, should all average out to something that is, roughly, 0.

In contrast, the Romer version of Taylor's model does not have that problem. There are two changes.
First, prices set at t are based on *current information.*

$$x_t = \tfrac{1}{2}[\phi\, m_t + (1 - \phi)\ p_t]$$

$$+ \tfrac{1}{2}\ [\phi\, E_t\, m_{t+1} + (1 - \phi)\ E_t\, p_{t+1}].$$

Where does the random term enter the pricing equation. It enters because at time t m_t is known. In David Romer's random walk model

$$m_t = m_{t-1} + u_t$$

The u_t in the x_t equation yields a random term in the pricing equation and results in an autocovariance of the x_t terms.

For this reason the Romer version is better than the Taylor version.

Remark 6. **Another mistake in the Taylor model is that the money supply rule is:**

$$m_t = (1 - \beta)\, w_t.$$

As a result the money supply rule depends on *current* information.

If Taylor had made the money supply rule depend on t-1 information he would have made:

$$m_t = (1 - \beta)\, w_{t-1}$$

In this case, however, the difference equation we have to solve would be *third* order. And that is much messier to solve analytically.

Alternatively, Ben Bernanke and his fellow governors might look at *last period's new price* (or wage): $x_{t-1.}$

In that case

$$m_t = (1 - \beta)\, x_{t-1}.$$

That might be a better model.

NOTE: David Romer talks about the Fischer model of Stanley Fischer. That model has staggered contracts, but not in an interesting way since monetary policy is stabilizing only because the monetary authority has more information than the price setters.

Remark 7. **There is one more peculiarity to the Taylor model.**

The choice of *β,* the money supply rule*,* gives a trade-off between the *stability* of income and the *stability* of the price level*.*

σ_w

σ_y

As β rises from 0 the variance of y rises and the variance of the price level falls. Therefore in a certain sense there is a tradeoff between *price stability* and *income stability.*

That sounds like a Phillips Curve.

FOOTNOTE:

$$w_t = ½ (x_t + x_{t-1})$$

$$= ½ [\alpha_1 x_{t-1} + \epsilon_t + x_{t-1})$$

$$= ½ [(1 + \alpha_1)x_{t-1} + \epsilon_t]$$

$$\sigma^2(w) = .25 (1 + \alpha_1)^2/(1 - \alpha_1^2) \sigma_\epsilon^2 + .25 \sigma_\epsilon^2$$

$$= .25 (1 + \alpha_1)/(1 - \alpha_1) \sigma_\epsilon^2 + .25 \sigma_\epsilon^2$$

which is less than zero, since $d\alpha/d\beta < 0$ for β sufficiently small.

$$\sigma^2(y) =. 5\beta^2 [1/(1 - \alpha_1)] \sigma_\epsilon^2 + \sigma_v^2$$

(omitting the sub 1 on the α for notational convenience)

for β sufficiently small.
END FOOTNOTE

However, the usual trade-off in *macro models* is not between *output* and the *price level* but between *output* and the *level of inflation.* It is indeed odd then to have a trade-off between variance in *output* and variance in the *price level.*
Nothing stops us then from using our model to calculate then how the *variance in inflation* will change with the *variance in output*, on the idea that greater *instability in inflation* leads to greater *instability in output*.

It turns out, however, that as β varies the variability in *inflation* is *exactly constant.* So there is no tradeoff here between the stability of inflation and the stability of output, which would be analogous to a Phillips Curve, where there is a trade off between the level of inflation and the level of output.

FOOTNOTE

Inflation = $(w_t - w_{t-1}) = .5\ (x_t - x_{t-2})$

$$x_t = \alpha^2 x_{t-2} + \epsilon_t + \alpha\, \epsilon_{t-1}$$ **(omitting the sub 1 on the α for notational convenience)**

So $$\sigma^2 (w_t - w_{t-1}) = \sigma^2(-[1-\alpha^2]\, x_{t-2} + \epsilon_t + \alpha\, \epsilon_{t-1})$$

$$= (1 - \alpha^2)^2/(1 - \alpha^2)\, \sigma_\epsilon^2 + \sigma_\epsilon^2 + \alpha^2 \sigma_\epsilon^2$$

$$= 2\, \sigma_\epsilon^2$$

Thus $\sigma^2 (w_t - w_{t-1})$ does not vary with β!

Remark 8. **There is an important paper by Larry Ball, which makes a similar point which I will describe.**

Ball considers what happens in a continuous time version of the Taylor model when the money supply *growth* rate changes.

In his model there is a uniform distribution of firms changing prices over the year.

Consider what happens if a steady-state with constant prices is disturbed.

Suppose the monetary authorities have decided to switch from a 0 growth of money to a negative growth of money.
Then the firms that are setting their prices *initially* see relatively little change in the money supply, but over the period that their prices are to be fixed, their *competitors will have considerably lower prices.*

To compete with their competitors they reduce their current prices considerably, even though the money supply has only *begun* to drop.

As a result the money supply rises relative to the price level.
Since the determinant of aggregate demand in the Taylor model is real balances or M/p, *real income* will rise.

Thus the Taylor model with rational expectations says that a decrease in money growth is likely to lead to an *increase* in income and *a decrease* in inflation.

Now, no one believes that.

For example, in the 1982-1983 recession, known to some as the Volcker Recession, there was a very large decrease in monetary growth, but also a very large *decrease* in income. Thus this finding of the model is counterfactual.

This is a case in which the forward looking rational expectations is important. If inflationary expectations were backward and adaptive then the price setters would not decrease their prices as much, and the model would not predict such peculiar results.

***Remark 9.* Taylor's model has money illusion because the prices are constant in nominal terms over the two-period interval.**

Later we will explore a sense in which the *contracts* in this model are *near rational* in the sense that agents' losses relative to full rationality are small.

***Remark 10.* It was later discovered by Calvo that staggered contracts could be represented much easier. Instead of having the alternating price-setting, as in our model, it was more parsimonious to have a continuum of firms, each of which set its price and then had a constant probability of changing its price per period of time. Maury put Calvo on his part of the reading list and may mention it, at least in passing.**

So the hazard function for the firm to make a change in nominal prices is the negative exponential, $\delta e^{-\delta h}$, where h is the length of time between setting its price and getting a new opportunity that it can re-set it.

Remark 11. Taylor's model has the advantage for this part of the course that it is a very good tool for teaching you how to use ARMA processes, and I think that that is a very general modeling technique to know since you can get results you want surprisingly cheaply.

<Questionnaire regarding section times>.

Return logs.

Collect logs.