Midterm

Economics 202 A

Fall 2003

1. **Monetary Policy**

   A certain economy is described by the following aggregate demand equation:

   $$y_t = m_t - p_t$$

   The money supply rule followed by the monetary authority is given by a random walk process:

   $$m_t = m_{t-1} + \epsilon_t,$$

   where ϵ_t is an independent and identically distributed error term with zero mean.

   Assume that the equilibrium price level in that economy is given by the following process:

   $$p_t = \alpha m_{t-1} + (1-\alpha)p_{t-1} + \eta_t,$$

   where $0 \leqslant \alpha \leqslant 1$ and η_t is an independent and identically distributed error term with zero mean. Assume that the process followed by ϵ_t is independent to the process followed by η_t.

   (a) Write y_t as a function of m_{t-1}, p_{t-1} and error terms.

   (b) Show that y_t follows an AR(1) process. For what values of α will the process be stationary?

   (c) Assuming potential output is normalized to zero, the Lucas aggregate supply curve is given by:

   $$y_t = [p_t - E_{t-1}(p_t)] + u_t$$

   Can this equation be consistent with the economy described above? Do you need any assumption about α for your answer?

2. **Rational Expectations**

   True, False, or Uncertain? Justify your answer.

   "If individuals form their expectations rationally and there is no money illusion, only unpredictable monetary supply changes can affect real activity"

3. **Unemployment**

   The crucial assumption of efficiency wage theory is that the worker's effort level, e, is positively related to the real wage level, w:

   $$e = e(w), where e^{'}(w) > 0$$

   Assume that the unique input in the production function is now effective labor, so that the production function relating the number of goods produced, Q, to the number of effective labor units employed, eL, is given by:

   $$Q = F(eL), where F'(eL) > 0 and F^{''}(eL) < 0$$

   (a) Set the firm's profit maximization problem and find the first order conditions.

   (b) Derive the Solow Condition.

   (c) Suppose that the function relating effort to the real wage is of the form:

   $$e(w) = A + Bw^{\gamma},$$

   where A, B and γ are parameters. Assume $A < 0$, $B > 0$ and $0 < \gamma < 1$. What real wage would the profit maximizing firm ideally pay?

   (d) Assume now that A is positive. What real wage would the profit maximizing firm ideally pay?

4. **Shift in Consumption Demand in the Dornbusch Model**

   Consider a small open economy. Suppose the demand for money in that economy is given by:

   $$m - p = \phi y - \lambda r$$

   Output y is constant. The uncovered interest rate parity equation holds:

   $$r = r^* + \dot{e}$$

where $\dot{e}$ is the rate of depreciation of the foreign currency, which is related to the foreign currency value by the following equation:

$$\dot{e} = \theta(\overline{e} - e)$$

The rate of change of prices, $\dot{p}$, depends on excess demand as follows:

$$\dot{p} = \pi\left[\mu + \delta(e - p) + (\gamma - 1)y - \sigma r\right]$$

(a) Represent the long run values of p and e in the pxe plane.

(b) What are the effects of a previously unanticipated expansion of consumption demand?

Suggested Answers to Midterm

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Econ 202 A Fall 2003

1. The economy is described by the following three equations:

$$y_t = m_t - p_t, \tag{1}$$

$$m_t = m_{t-1} + \epsilon_t, \tag{2}$$

$$p_t = \alpha m_{t-1} + (1-\alpha)p_{t-1} + \eta_t, \tag{3}$$

where $0 \leqslant \alpha \leqslant 1$ and ϵ_t and η_t are independent white noise processes.

(a) Plugging equations (2) in (1):

$$\begin{aligned} y_t &= m_t - p_t \\ &= m_{t-1} + \epsilon_t - p_t \end{aligned}$$

Using equation (3), we can rewrite the above expression as follows:

$$\begin{aligned} y_t &= m_{t-1} + \epsilon_t - \alpha m_{t-1} - (1-\alpha)p_{t-1} - \eta_t \\ &= (1-\alpha)m_{t-1} - (1-\alpha)p_{t-1} + \epsilon_t - \eta_t \\ &= (1-\alpha)\left(m_{t-1} - p_{t-1}\right) + \epsilon_t - \eta_t \end{aligned}$$

(b) Backwarding equation (1) by one period, we have:

$$y_{t-1} = m_{t-1} - p_{t-1}$$

Thus, the result obtained in part (a) can be rewritten as follows:

$$\begin{aligned} y_t &= (1-\alpha)\left(m_{t-1} - p_{t-1}\right) + \epsilon_t - \eta_t \\ &= (1-\alpha)y_{t-1} + \epsilon_t - \eta_t \end{aligned}$$

The equation above shows that y_t is a function of its first order lag, y_{t-1}, and a random term $\epsilon_t - \eta_t$. To show that y_t follows an AR(1) process, it remains to show that $\epsilon_t - \eta_t$ is a white noise process.

For simplicity, call $u_t = \epsilon_t - \eta_t$. Applying the expectations operator on both sides of this identity, we obtain:

$$E\left(u_t\right) = E\left(\epsilon_t\right) - E\left(\eta_t\right) = 0 - 0 = 0$$

Squaring both sides of the identity and applying again the expectations operator on both sides, we obtain:

$$\begin{aligned} E\left(u_t^2\right) &= E\left[\left(\epsilon_t - \eta_t\right)^2\right] \\ &= E\left(\epsilon_t^2\right) + E\left(\eta_t^2\right) - 2E\left(\epsilon_t \eta_t\right) \\ &= \sigma_\epsilon^2 + \sigma_\eta^2 - 0 \\ &= \sigma_\epsilon^2 + \sigma_\eta^2 = \sigma_u^2 \end{aligned}$$

Finally, multiplying u_t by u_{t-s}, where $s \neq 0$, and taking expectations, we obtain:

$$\begin{aligned} E\left(u_t u_{t-s}\right) &= E\left[\left(\epsilon_t - \eta_t\right)\left(\epsilon_{t-s} - \eta_{t-s}\right)\right] \\ &= E\left(\epsilon_t \epsilon_{t-s}\right) - E\left(\epsilon_t \eta_{t-s}\right) - E\left(\eta_t \epsilon_{t-s}\right) + E\left(\eta_t \eta_{t-s}\right) \\ &= 0 - 0 - 0 + 0 = 0 \end{aligned}$$

Hence, because $E\left(u_t\right) = 0$, $E\left(u_t^2\right) = \sigma_u^2$ (constant), and $E\left(u_t u_{t-s}\right) = 0$, for $s \neq 0$, u_t follows a white noise process and the process $y_t = (1-\alpha)y_{t-1} + u_t$ is indeed an AR(1) process.

The process will be stationary if and only if $|1 - \alpha| < 1$. That is, stationarity will occur if and only if $-1 < 1 - \alpha < 1$ or, equivalently, $0 < \alpha < 2$. Because $0 \leqslant \alpha \leqslant 1$, the process will be stationary for any value of α, except $\alpha = 0$. Thus, for $\alpha \in (0, 1]$ the process will be stationary.

(c) A key implication of the Lucas aggregate supply curve is that the expected value of output has to be equal to the expected value of potential output (which is zero, by assumption):

$$\begin{aligned} E_{t-1}\left(y_t\right) &= E_{t-1}\left[p_t - E_{t-1}\left(p_t\right)\right] + E_{t-1}\left(u_t\right) \\ &= E_{t-1}\left(p_t\right) - E_{t-1}\left(p_t\right) + 0 \\ &= 0, \end{aligned}$$

where we used the fact that $E_{t-1}(p_t) = E_{t-1}[E_{t-1}(p_t)]$ and $E_{t-1}(u_t) = 0$. Thus, to be consistent with a Lucas aggregate supply in which potential output is normalized to zero, the economy has to satisfy the condition $E_{t-1}(y_t) = 0$. By our result in part (a):

$$\begin{aligned} E_{t-1}(y_t) &= (1-\alpha)E_{t-1}(y_{t-1}) + E_{t-1}(\epsilon_t) - E_{t-1}(\eta_t) \\ &= (1-\alpha)y_{t-1} \end{aligned}$$

Now, we can easily see that the hypothetical economy above will only be consistent with the Lucas aggregate supply curve if $\alpha = 1$.

2. True. With rational expectations and no money illusion, prices fully reflect the information available at each point in time (no price stickiness is present). In particular, the predictable part of money supply will be incorporated in the price setting process and, consequently, only unpredictable changes will affect output.

3. (a) Firm's profit is given by $\pi = F(e(w)L) - wL$. Hence, the firm's profit maximization problem is the following:

$$\max_{w,L} F(e(w)L) - wL$$

The first order condition with respect to w is given by:

$$\begin{aligned} \frac{\partial \pi}{\partial w} &= 0 \iff F'(e(w)L) \cdot e'(w)L - L = 0 \\ &\iff F'(e(w)L) \cdot e'(w) = 1 \end{aligned} \tag{4}$$

The first order condition with respect to L is given by:

$$\begin{aligned} \frac{\partial \pi}{\partial L} &= 0 \iff F'(e(w)L) \cdot e(w) - w = 0 \\ &\iff F'(e(w)L) \cdot e(w) = w \end{aligned} \tag{5}$$

(b) The Solow condition follows from the two first order conditions derived in part (a). From equation (4), $F'(e(w)L) = \frac{1}{e'(w)}$. Plugging this in equation (5), we obtain $\frac{e(w)}{e'(w)} = w$, which can be rewritten in the following form:

$$\frac{e'(w) \cdot w}{e(w)} = 1$$

This equation id the so-called *Solow Condition*. It states that the wage the firm would ideally set in the optimum is the one that makes the wage elasticity of effort equal to 1.

(c) The wage that firms would ideally pay (that is, the wage they pay when they are not constrained by the supply of labor) is implicitly given by the Solow Condition derived in part (c).Assuming the effort function is given by $e(w) = A + Bw^{\gamma}$, we have:

$$\begin{aligned}
\frac{e'(w) \cdot w}{e(w)} &= 1 \Longleftrightarrow \frac{\gamma B w^{\gamma-1} \cdot w}{A + Bw^{\gamma}} = 1 \\
&\Longleftrightarrow \gamma B w^{\gamma} = A + Bw^{\gamma} \\
&\Longleftrightarrow w^{\gamma} = \frac{-A}{B(1-\gamma)} \\
&\Longleftrightarrow w = \left[\frac{-A}{B(1-\gamma)}\right]^{\frac{1}{\gamma}}
\end{aligned}$$

(d) When $A > 0$, the wage that firms would set according to the Solow Condition is negative. Since this is infeasible, they set a wage of zero. Note that, even when setting a wage of zero, firms obtain a positive effort: $e(0) = A > 0$. So, they will have no incentive to pay a positive wage.

4. Dornbush's model can be described by the following system of equations:

$$m - p = \phi y - \lambda r \tag{6}$$

$$r = r^* + \dot{e} \tag{7}$$

$$\dot{e} = \theta(\bar{e} - e) \tag{8}$$

$$\dot{p} = \pi\left[\mu + \delta(e-p) + (\gamma - 1)y - \sigma r\right] \tag{9}$$

(a) In the long run, we will have: $\dot{p} = 0$, $\dot{e} = 0$, $p = \bar{p}$, $e = \bar{e}$, $r = r^*$. Thus, in the long run, equation (9) can be rewritten as follows:

$$\bar{p} = \bar{e} + \frac{\mu}{\delta} + \left(\frac{\gamma - 1}{\delta}\right) y - \frac{\sigma}{\delta} r^* \tag{10}$$

This equation shows that $\bar{p}$ and $\bar{e}$ have a one-to-one relation. Hence, in the plane pxe the long-run values $\bar{p}$ and $\bar{e}$ have to be represented over a 45 degrees line with intercept given by $\frac{\mu}{\delta} + \left(\frac{\gamma-1}{\delta}\right) y - \frac{\sigma}{\delta} r^*$. To find the exact combination of $\bar{p}$ and $\bar{e}$ that will hold in equilibrium, we need an extra equation. For that, note that equation (6) in the long run becomes:

$$m - \bar{p} = \phi y - \lambda r^* \tag{11}$$

Subtracting (6) from (11), and then using equations (7) and (8), we obtain:

$$p - \bar{p} = \lambda (r - r^*) = \lambda \dot{e} = \lambda\theta(\bar{e} - e)$$

which can be rewritten as:

$$p = \bar{p} + \lambda\theta(\bar{e} - e) \tag{12}$$

Equation (12) is downward sloping in e and shows how the price level and the exchange rate are related with their long run values. In particular, note that this equation passes through the point $(\bar{p}, \bar{e})$. Hence, the long run values $\bar{p}$ and $\bar{e}$ can be represented in the plane pxe as the intersection of equations (10) and (12). See Figure 1.

(b) An expansion in consumption demand can be represented by a positive increase in the parameter μ. From equation (11), we see that $\bar{p}$ is unaffected by the increase in μ. Thus, by equation (10), we can see that $\bar{e}$ decreases to counteract the shift in μ. Because μ increases, $\bar{e}$ decreases and $\bar{p}$ stays the same, the lines that represents equation (10) and (12) in the previous graph will both shift to the left. And this shifts will be such that the price level will keep fixed at $\bar{p}$. See Figure 2.

Because the economy always has to be over the line given by (12), there will be no dynamics. Just after the unanticipated shift in μ, the exchange rate will jump to its new long run value and the economy will be immediately in its new long run equilibrium.

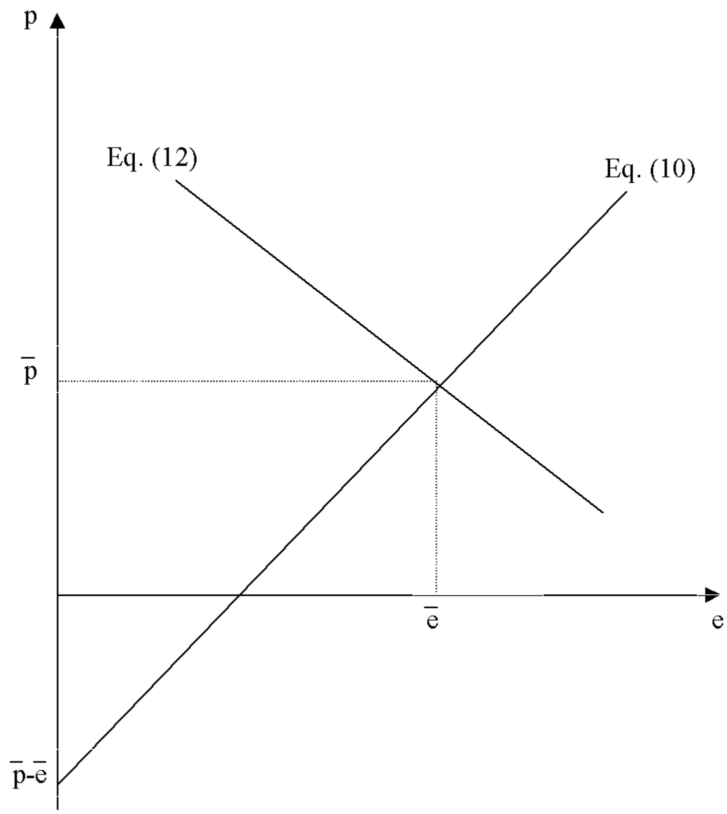

Figure 1: Problem 4, part (a)

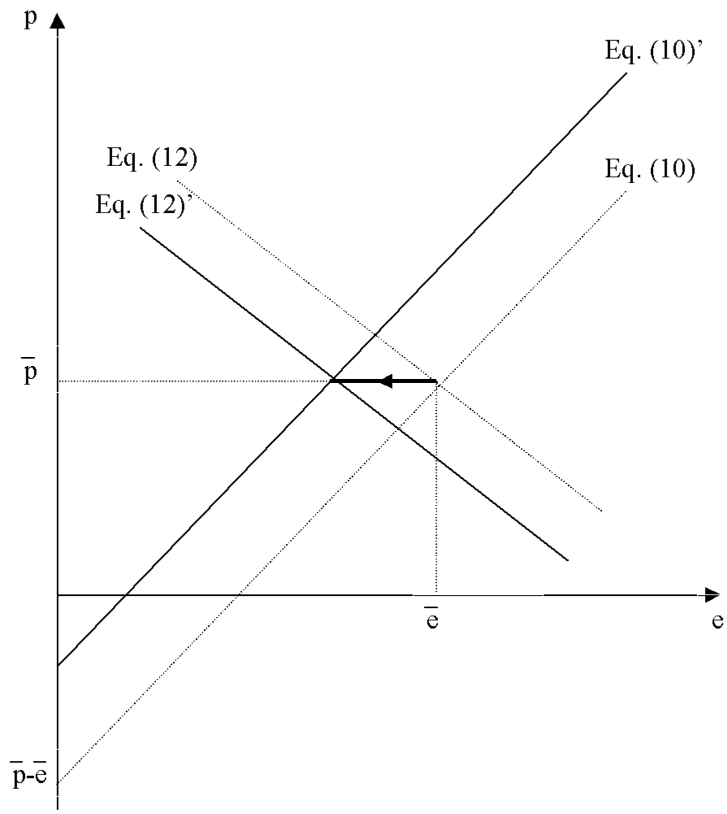

Figure 2: Problem 4, part (b)