Midterm Exam

Economics 202A Fall 2004

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Please answer all four questions.

1. Consider the following economy

(1)
$$p_t = m_t - a y_t + u_t$$

(2)
$$y_t = b(p_t - p_t)$$

(3) $m_t = \alpha m_{t-1} + v_t$

where

u, is MA (1)

 $u_t = \epsilon_t + \beta \epsilon_{t-1}$

and ϵ_t is i.i.d. N(0, σ_{ϵ}^2),

and where v_t is N(0, σ_v^2).

(i). Identify which of these equations is an aggregate demand cure, an aggregate supply curve, and the money supply rule, according to the standard model with standard notation.

(ii.) What is the expected price level at time *i.e.*, $p_t^* = E(p_t | \theta_{t-1})$, given rational expectations. Express p_t^* in terms of current and past shocks.

2. Consider the following numerical variant of the Dornbusch model. The demand for money is given by

$$m^{d}/p = y^{\frac{1}{2}} r^{\frac{1}{2}}$$
.

The "Phillips Curve" is given by

 $dp/dt = \frac{3}{4} [3 + .5 (e - p) + .5 y - .25 r].$

It is currently announced that at future date T the money supply will change discontinuously by 10 %.

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At time T, (i) by what percentage will the exchange rate change? (ii) by what percentage will the price level change? (iii) by what percentage will the interest rate change?

3. Hamlet lives for two periods (periods one and two).

His intertemporal utility function is

 $U = 24 c_1 - c_1^2 + \frac{1}{2} (24 c_2 - c_2^2).$

His income in period 1 is 6. His income in period 2 is uniformly distributed between 3 and 9. The rate of interest is 0.

What is Hamlet's period 1 consumption?

4. Ophelia is considering buying a plot of land on which she can grow a single tree. The cost of planting, harvesting and *replanting a tree after harvest* are all zero. A tree can be cut down at annual intervals and a new tree planted in its place at that time. At age one year it yields \$10. at age two years it yields \$36, and at age three years, it yields \$77. After that the tree dies. The rate of interest is 100 % per year.

What is the maximum price that Ophelia should pay for the plot of land?

[Hint: use recursion to answer this problem.]

Midterm Exam's Answer Key University of California at Berkeley Econ 202A, Macro Theory Fall 2004

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1. (i)

(1) aggregate demand

(2) aggregate supply

(3) money supply rule

(ii) $E[y_t|\theta_{t-1}] = b[E[p_t|\theta_{t-1}] - E[p_t|\theta_{t-1}]] = 0$

 $\mathrm{E}[p_t|\boldsymbol{\theta}_{t-1}] = \mathrm{E}[m_t|\boldsymbol{\theta}_{t-1}] + \mathrm{E}[u_t|\boldsymbol{\theta}_{t-1}] = \alpha m_{t-1} + \beta \varepsilon_{t-1} = \sum_{i=1}^{\infty} \alpha^i v_{t-i} + \beta \varepsilon_{t-1}$

Note: the last equality follows from the fact that an AR(1) can be expressed as an MA(∞) under the usual stationarity condition, $|\alpha| < 1$.

2. (i) the exchange rate, e, does not change at time T.

(ii) the price level, p, does not change at time T.

(iii) Since the money supply increases by 10%, the interest rate must adjust to clear the money market. Note that we can rewrite the money market condition in logarithmic form so that:

 $\log m - \log p = 1/2\log y - 1/2\log r$

From this, it is easy to see that $dm/dr^*(r/m) = -1/2$. Hence if the money supply increases by 10% at time T, the interest rate must fall by 10%/1/2 = 20%.

Note: It is also a solution if you derive the exact change in r. This is 21%. The difference is due to the fact that the above solution is an approximation to the actual change in r.

3. The Euler condition for this problem is $u'(c_1) = 1/2[E[u'(c_2)]]$, or since utility is quadratic and so marginal utility is linear: $u'(c_1) = 1/2[u'(E[c_2])]$.

Thus,
$$24 - 2c_1 = \frac{1}{2}[24 - 2E[c_2]]$$

 $\Rightarrow 24 - 2c_1 = 12 - E[c_2]$
(1)
 $r = 0 \Rightarrow c_1 + c_2 = y_1 + y_2 \Rightarrow c_1 + E[c_2] = y_1 + E[y_2]$
 $y_2 \sim U[3,9] \Rightarrow E[y_2] = 6 \Rightarrow c_1 + E[c_2] = 12 \Rightarrow E[c_2] = 12 - c_1$
(2)
Substituting (2) into (1) yields:

Substituting (2) into (1) yields:

 $24 - 2c_1 = 12 - (12 - c_1) = c_1 \Rightarrow c_1 = 8.$

4. There are 3 strategies conditional on owning the plot of land:

One can harvest every year, every two years or every three years. Let V_i denote that value of harvesting after the ith year. Ophelia should be willing to buy the land for at most $V = \max{\{V_1, V_2, V_3\}}$.

 $V_1 = 10/2 + V_1/2 \Rightarrow V_1 = 10$ $V_2 = 36/4 + V_2/2 \Rightarrow V_2 = 12$ $V_3 = 77/8 + V_3/8 \Rightarrow V_3 = 11$

Hence V = 12.