Department of Economics **Economics 202A** **Fall 2006**
University of California, Berkeley **Akerlof/Halac**

There are three questions. Answer all parts to all questions.

1. Inventory-Sales Problem [40 points]

You hold an inventory of one bushel of wheat. The bushel costs you *s* to store per period. The price per bushel, *p*, randomly varies over time and is *i.i.d.* Your goal is to sell the wheat to maximize your discounted profits.

Assume that the discount rate for this problem is 0.

A. Explain why, if a price *p* is observed in the first period, the current value of the bushel of wheat, is:

$$V(p) = \max \{p, [-s + E\, V(p')]\},$$

where V(p´) is the value of the wheat, conditional on observing a price p´ one period from now. E represents expectations.

Using intuition, explain why the solution to this problem is a "threshold rule":

sell, if $p > p^*$

wait, if $p \leq p^*$.

Also, explain why the threshold rule satisfies the equality:

$$p^* = -s + E\, V(p).$$

B. Assume that *p* is distributed uniformly between 0 and 1, and that s≤½.

Show that:

$$p^* = 1 - [(1 - (1 - 2s)]^{½}.$$

What is the value of p* as the cost of storage, *s*, approaches zero? Give an intuitive explanation for your answer.

2. A Taylor Model [40 points]

Consider an economy with two competing firms, $i = \{A, B\}$, whose prices are fixed over one-year intervals. Firm *A* sets its prices in January and Firm *B* sets its prices in July. Let *t* be six-month periods, so that *A* sets its prices at $t = 0, 2, 4$, etc. and *B* at $t = 1, 3, 5$, etc.

Assume that the ideal price for firm *i* at time *t* is equal to the money supply at time *t*:

$$p^*_{it} = m_t$$

The price set by a firm at time *t*, x_t, is an average of the ideal price at *t* and the expected ideal price at $t + 1$ (with the expectation taken at time *t*):

$$x_t = \tfrac{1}{2}\, p^*_{it} + \tfrac{1}{2}\, E_t\, p^*_{it+1}$$

Regarding expectations, the firm setting the price at time *t* knows the money supply at that time.

The aggregate price level is:

$$p_t = \tfrac{1}{2}\, x_{t-1} + \tfrac{1}{2}\, x_t$$

The aggregate demand curve is:

$$y_t = m_t - p_t$$

A. Suppose that initially the money supply m_t is known to be 1; it has always been 1; and it is expected to remain at that level. What is the level of output at this long-run equilibrium?

B. Suppose that for all times prior to time 1, $m_t=1$ as in part (a). At time 1, the money supply is unexpectedly increased permanently, from 1 to 2. It is then expected to remain at 2 for the rest of time. (The price-setter at time 1 knows that the money supply at that time has increased from 1 to 2.)

Calculate y_1. What is the impact on output? Calculate y_t for $t>1$ and show that these effects are only transitory.

C. In part B you showed that in this model monetary policy does not have long-lasting effects on output. In the original Taylor model, however, changes in the money supply have persistent effects on output. Both models assume unsynchronized price setting and prices fixed over two-period intervals. What is then different in this model that makes the effects of monetary policy *not* persistent? Explain briefly.

3. True, False, Uncertain [20 points].

Identify whether both of the following two statements, are True, False or Uncertain. Explain your answer.

Statement 1.

"Edmund S. Phelps, a Columbia University professor, was awarded the Nobel Memorial Prize in Economic Science yesterday for his contribution to a sophisticated explanation of how wages, unemployment and inflation interact with one another. The explanation holds, in essence, that wages and inflation tend to rise in tandem, one pushing up the other, until the unemployment rate reaches an 'equilibrium' or 'natural' level at which prices no longer rise." ***New York Times,*** **Tuesday, October 10.**

Statement 2.

"The Shapiro-Stiglitz shirking model predicts that the bulk of those unemployed at any time are those who were fired for shirking."

Suggested Answers to Midterm 2006

1 Inventory-Sales Problem

(A) The expression for the value of the bushel of wheat uses recursion. In the first period, we have two options: (i) we can sell the bushel today at an observed price p; or (ii) we can wait until next period, pay the storage cost s, and then face exactly the same situation with a new observed price p'. As this is repeated every period, we have a recursive structure. The expected value if we choose to wait is then the expected value of the bushel conditional on observing a price p' next period, $EV(p')$, minus the cost of storage, s. Since to maximize profits we choose the best of the two options, the value of the bushel is the maximum of the two values.

Since the price is an *i.i.d.* random variable, the solution to this problem is a simple threshold rule. If the price observed today, p, is larger than the expected value of the bushel net of storage costs, $-s + EV(p)$, then it is profitable to sell the bushel today. If $p \leqslant -s + EV(p)$, we should wait because the expected value of the bushel is larger than the price observed today, and it more than compensates for storage costs. The relevant threshold is then $p^* = -s + EV(p)$.

(B) Since $p^* = -s + EV(p)$,

$$p^* + s = EV(p) = E\ \max\{p, p^*\} = \int_0^{p^*} p^* dp + \int_{p^*}^1 p\ dp = p^{*2} + \frac{1}{2} - \frac{p^{*2}}{2} = \frac{p^{*2}}{2} + \frac{1}{2}$$

Therefore,

$$\frac{p^{*2}}{2} - p^* + \left(\frac{1}{2} - s\right) = 0$$

which gives

$$p^* = 1 \pm \sqrt{1 - 2\left(\frac{1}{2} - s\right)} = 1 \pm \sqrt{1 - (1 - 2s)}$$

and since $p \sim U[0, 1]$,

$$p^* = 1 - \sqrt{1 - (1 - 2s)}$$

As s approaches 0, p^* approaches 1. Intuitively, if $s = 0$, then there is no cost of waiting, since the cost of storing the bushel is 0 and the discount rate is also 0. Therefore, we should not sell the bushel unless we observe a price p equal to the maximum possible price, which is 1.

2 A Taylor Model

(A) If m_t has always been equal to 1 and it is expected to remain at 1,

$$x_t = \frac{1}{2}p^*_{it} + \frac{1}{2}E_t p^*_{it+1} = \frac{1}{2}m_t + \frac{1}{2}E_t m_{t+1} = 1 \quad \forall t$$

Hence,

$$\begin{aligned} p_t &= \frac{1}{2}x_{t-1} + \frac{1}{2}x_t = 1 \\ y_t &= m_t - p_t = 0 \end{aligned}$$

(B) For $t = 0, 1$, note that x_t is:

$$\begin{aligned} x_0 &= \frac{1}{2}m_0 + \frac{1}{2}E_0 m_1 = 1 \\ x_1 &= \frac{1}{2}m_1 + \frac{1}{2}E_1 m_2 = 2 \end{aligned}$$

Hence,

$$\begin{aligned} p_1 &= \frac{1}{2}x_0 + \frac{1}{2}x_1 = \frac{3}{2} \\ y_1 &= m_1 - p_1 = \frac{1}{2} \end{aligned}$$

The increase in the money supply has an expansionary effect on output. But this effect is only transitory. After $t = 1$ output goes back to its long-run equilibrium level of 0. For $t > 1$:

$$\begin{aligned} x_t &= \frac{1}{2}m_t + \frac{1}{2}E_t m_{t+1} = 2 \\ p_t &= \frac{1}{2}x_{t-1} + \frac{1}{2}x_t = 2 \\ y_t &= m_t - p_t = 0 \end{aligned}$$

(C) The difference is that here the ideal price of the firm does not depend on the competitor's price as in the Taylor model. When setting prices, firms do not take into account the other firm's price. As a result, firms will adjust fully to changes in the money supply, even though the other firm cannot adjust its price until the next period. This then eliminates one source of nominal rigidity that is present in the Taylor model, and makes the effects of monetary policy not persistent.

3 True, False, Uncertain

Statement 1

False. Recall the expectations-augmented Phillips curve formulated by Phelps:

$$\pi_t = \pi_t^e + \beta(U^* - U_t),$$

where π_t^e is the expected inflation rate at time t and U^* is the natural rate of unemployment. With expectations based on observations of inflation in the recent past, it follows that inflation will be increasing as long as $U_t < U^*$ and decreasing when $U_t > U^*$. In steady state, $U_t = U^*$ and the inflation rate is constant ($\pi_t = \pi_t^e$). However, the inflation rate needs not be equal to zero in steady state. The statement is thus False because it says that wages and inflation tend to rise in tandem until the unemployment rate reaches an equilibrium level "at which prices no longer rise," whereas it should say "at which inflation no longer rises."

Statement 2

False. In equilibrium, as the threat associated with being fired is effective, no shirking will actually occur in the Shapiro-Stiglitz model. The unemployed are a rotating pool of individuals who have quit jobs for personal reasons, who are new entrants to the labor market, or who have been laid off by firms with declines in demand.