Economics 202A
MIDTERM EXAM

Instructions:

1. Be sure to write your name on the cover of each Blue Book.

2. The questions differ in difficulty but count equally. Each question counts 20 points for a total of 80 for the whole exam.

3. Answer all parts to all questions.
1. Show formally from Friedman’s model of consumption that there is a positive correlation between transitory and current income.

2. Suppose that $D_t = \alpha e_{t-5} + e_t$ where the $e_t$’s are i.i.d. $N(0, \sigma_e^2)$. What is $E_t(D_{t+3})$?

3. Show that in Mankiw’s model (in the absence of a menu cost $z$) the loss from failure to change price after a constant shift in demand is equal to $2C$ (where $C$ is the area of the small triangle in Mankiw’s key diagram).

   [Definitions and hints: If $(q^m, p^m)$ are the maximizing quantity and price and $(q^n, p^n)$ are the non-maximizing quantity and price, $C$ is by definition $\frac{1}{2} (q^n - q^m)(p^m - p^n)$. To answer this question you must specify Mankiw’s model, recall what $C$ is, and show that it is equal to $\frac{1}{2}$ the loss in profits.]

4. Consider a firm $i$ that minimizes at time $t$:

   $$E_t \sum_{j=0}^{\infty} \left( \frac{1}{1+r} \right)^j \left[ (p_{i,t+j} - p^*_{i,t+j})^2 + c(p_{i,t+j} - p_{i,t+j-1})^2 \right], \quad c > 0,$$

   where $p_{i,t+j}$ is the log of the nominal price of firm $i$ in period $t+j$, and $p^*_{i,t+j}$ is the log of the nominal price that firm $i$ would choose in period $t+j$ in the absence of adjustment costs. Costs of changing nominal prices are captured by the second term in the objective function. The information set at time $t$ includes current and lagged $p_{i,t}$ and $p^*_{i,t}$.

   a. Derive the first order condition of the above minimization problem, giving the current price $p_{i,t}$ as a function of itself lagged, of its expectation at $t+1$, and of the current optimal price.

   b. Rewrite the first order condition using the lag operator. Solve by factorization to derive the following expression:

   $$p_{i,t} = \frac{1}{\lambda_1} p_{i,t-1} + \frac{1+r}{\lambda_2} c \sum_{j=0}^{\infty} \left( \frac{1}{\lambda_2} \right)^j E_t p^*_{i,t+j},$$

   where $\lambda_1$ and $\lambda_2$ are the reciprocals of the roots of $(1+r)x^2 - \left(\frac{1+r}{c} + 1 + r + 1\right)x + 1 = 0$ and $\lambda_1$ is the smaller of the two. Interpret your result.
Suggested solutions to the midterm exam

Andrea De Micheli

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1. Recall that in Friedman’s model of consumption, we assume \( y_C = y_P + y_T \) and \( \rho(y_P, y_T) = 0 \) so that:

\[
\text{Cov}(y_C, y_T) = \text{Cov}(y_P + y_T, y_T) = \text{Cov}(y_P, y_T) + \text{Cov}(y_T, y_T) = 0 + \text{Var}(y_T) > 0.
\]

Thus, there is positive correlation between current and transitory income.

2. As the \( \varepsilon_t \)’s are white noise, we have:

\[
E_t(\Delta_{t+3}) = E_t(\alpha \varepsilon_{t+3-5} + \varepsilon_{t+3}) = E_t(\alpha \varepsilon_{t-2}) + E_t(\varepsilon_{t+3}) = \alpha E_t(\varepsilon_{t-2}) + 0 = \alpha \varepsilon_{t-2}
\]

3. A monopolist facing linear demand and constant marginal cost maximizes:

\[
\pi = (a + \varepsilon) p - bp^2 - c(a + \varepsilon) + cbp.
\]

The FOC with respect to \( p \) yields:

\[
\frac{\partial \pi}{\partial p} = a + \varepsilon - 2bp + cb = 0
\]

\[
\Rightarrow p^m = \frac{a + \varepsilon + cb}{2b}.
\]

Plugging the above expression into the inverse demand function, we find:

\[
q^m = \frac{a + \varepsilon - cb}{2}.
\]

If the monopolist does not reoptimize after the shock, it will charge:

\[
p^n = p^m(\varepsilon = 0) = \frac{a + cb}{2b},
\]

and it will sell:

\[
q^n = q(p^n) = \frac{a + 2\varepsilon - cb}{2}.
\]

If the monopolist does not reoptimize, she will then loose:

\[
\text{LOSS} = \pi^m - \pi^n = ...
\]

\[
= (p^m - p^n)q^n - (p^n - c)(q^n - q^m) =
\]

\[
= A - B = ... =
\]

\[
= \frac{a \varepsilon + \varepsilon^2 - cb \varepsilon}{4b} = \frac{a \varepsilon - cb \varepsilon}{4b} =
\]

\[
= \frac{\varepsilon^2}{4b}.
\]
By definition $C$ is:

$$C = (p^m - p^n)(q^n - q^m) = ... = \frac{1}{2}v^2 \frac{\beta}{\delta}$$

Thus, we have shown that $LOSS = 2C$.

4. The firm solves:

$$\min_{(p_{i,t+j})_{j=0}^{\infty}} \mathbb{E}_t \sum_{j=0}^{\infty} \left( \frac{1}{1+r} \right)^j \left[ (p_{i,t+j} - \bar{p}_{i,t+j})^2 + c(p_{i,t+j} - p_{i,t+j-1})^2 \right]$$

(a) The FOC with respect to $p_{i,t}$ yields:

$$\Rightarrow \left( \frac{1}{1+r} \right)^0 \left[ 2(p_{i,t} - \bar{p}_{i,t}) + 2c(p_{i,t} - p_{i,t-1}) \right] + \mathbb{E}_t \left( \frac{1}{1+r} \right)^1 \left[ -2c(p_{i,t+1} - p_{i,t}) \right] = 0$$

$$\Leftarrow (p_{i,t} - \bar{p}_{i,t}) + c(p_{i,t} - p_{i,t-1}) - c \left( \frac{1}{1+r} \right) (\mathbb{E}_t (p_{i,t+1}) - p_{i,t}) = 0$$

$$\Leftarrow \left( 1 + c + \frac{c}{1+r} \right) p_{i,t} - \bar{p}_{i,t} - cp_{i,t-1} - \frac{c}{1+r} \mathbb{E}_t (p_{i,t+1}) = 0$$

(b) Using the lag operator, we can rewrite the FOC as:

$$\Leftarrow \left( 1 + c + \frac{c}{1+r} \right) p_{i,t} - \bar{p}_{i,t} - cLp_{i,t} - \frac{c}{1+r}L^{-1}p_{i,t} = 0$$

$$\Leftarrow \left( \left( 1 + c + \frac{c}{1+r} \right) - cL - \frac{c}{1+r}L^{-1} \right) p_{i,t} = \bar{p}_{i,t}$$

$$\Leftarrow \frac{1+r}{c}L \left( \frac{1+r+c+cr+c}{1+r} - cL - \frac{c}{1+r}L^{-1} \right) p_{i,t} = \frac{1+r}{c}Lp_{i,t}$$

$$\Leftarrow \left( \frac{1+r}{c} + 1 + r + 1 \right) L - (1+r)L^2 - 1 \right) p_{i,t} = \frac{1+r}{c}Lp_{i,t}$$

We can factor $\frac{1+r}{c} + 1 + r + 1 - (1+r)x^2 - 1$ as $-(1 - \lambda_1x)(1 - \lambda_2x)$ where

$$\lambda_1 + \lambda_2 = \frac{1+r}{c} + 1 + r + 1,$$

$$\lambda_1 \lambda_2 = 1 + r,$$

and therefore

$$(1 - \lambda_1)(1 - \lambda_2) = 1 - (\lambda_1 + \lambda_2) + \lambda_1 \lambda_2 = -\frac{1+r}{c}. \quad (1)$$

Since the sum and the product of $\lambda_1$ and $\lambda_2$ are positive, both roots are positive. Furthermore, equation (1) implies that one of the root is greater than one and while the other is smaller than one. Let $\lambda_1$ be the smaller root so that we have $0 < \lambda_1 < 1 < \lambda_2$.

Factorization then yields:

$$\Leftarrow -(1 - \lambda_1L)(1 - \lambda_2L)p_{i,t} = \frac{1+r}{c}Lp_{i,t}$$

$$\Leftarrow -(1 - \lambda_1L) \left( \frac{1 - \lambda_2L}{\lambda_2L} \right) p_{i,t} = \frac{1+r}{c} \lambda_2Lp_{i,t}$$
\[ -(1 - \lambda_1 L) \left( \frac{1}{\lambda_2} L^{-1} - 1 \right) p_{i,t} = \frac{1 + r}{c} \frac{1}{\lambda_2} p_{i,t}^* \]

\[ (1 - \lambda_1 L) \left( 1 - \frac{1}{\lambda_2} L^{-1} \right) p_{i,t} = \frac{1 + r}{c} \frac{1}{\lambda_2} p_{i,t}^* \]

\[ (1 - \lambda_1 L) p_{i,t} = \frac{1 + r}{c} \frac{1}{\lambda_2} \frac{1}{1 - \frac{1}{\lambda_2} L^{-1}} p_{i,t}^* \]

\[ (1 - \lambda_1 L) p_{i,t} = \frac{1 + r}{c} \frac{1}{\lambda_2} \sum_{j=0}^{\infty} \left( \frac{1}{\lambda_2} L^{-1} \right)^j p_{i,t+j}^* \]

where in the last line we used the fact that \( \frac{1}{\lambda_2} \) is inside the unit circle. At this point, we just need to use the lag operator to get the expression we are looking for:

\[ p_{i,t} = \lambda_1 p_{i,t-1} + \frac{1 + r}{c} \frac{1}{\lambda_2} \sum_{j=0}^{\infty} \left( \frac{1}{\lambda_2} \right)^j \mathbb{E}_t \left( p_{i,t+j}^* \right). \]

Therefore, the current price charged by the firm is a weighted average between the price in the past period and its expectation of the optimal price in the present and the future periods. (This problem was adapted from a paper by Julio Rotemberg published in the Review of Economic Studies in 1982.)