University of California, Berkeley
Department of Economics

Economics 230a
Fall 2004

Problem Set #1

(due 10/5/04)

1. Consider an economy in which relative producer prices are fixed and a representative household maximizes the following utility function in consumption goods and leisure (l):

$$U(c_1, c_2, l) = \beta_1 \log(c_1 - a_1) + \beta_2 \log(c_2 - a_2) + \beta_3 \log l$$

(where $\beta_1+\beta_2+\beta_3 = 1$), subject to the budget constraint:

$$p_1 c_1 + p_2 c_2 = w(1 - l)$$

A. Rewrite the household's problem in terms of consumption and labor supply, $L = 1-l$.

B. Derive an explicit expression for the excess burden of taxes on c_1, c_2, and L as a function of original prices, say p_1^0, p_2^0, w^0, distorted prices, p_1^1, p_2^1, w^1, and a fixed utility level.

C. Using the measure you derived in Part B, show that the deadweight loss is zero for a combined consumption tax/wage subsidy policy that raises the prices p_1, p_2, and w by the same proportion.

D. Starting from an initial price vector (p_1, p_2, w), consider the following two experiments:

1. a uniform tax on consumption that makes the final price vector ($p_1/(1-\theta)$, $p_2/(1-\theta)$, w)
2. a wage tax that makes the final price vector (p_1, p_2, $w(1-\theta)$)

Show that, for given utility, the deadweight losses for the two experiments are the same.

2. Consider an economy with fixed producer prices and a representative household that maximizes utility that is a function of one consumption good and two types of leisure, perhaps the leisure of two spouses,

$$U(c, l_1, l_2) = c^{\alpha_0} l_1^{\alpha_1} l_2^{\alpha_2}$$

(where $\alpha_0+\alpha_1+\alpha_2 = 1$), subject to the budget constraint:

$$pc = w_1(1 - l_1) + w_2(1 - l_2)$$

A. As in Problem #1, rewrite the household's problem in terms of consumption and labor supply, $L_1 = (1-l_1)$ and $L_2 = (1-l_2)$.

B. Derive expressions for the compensated and market supplies of L_1 and L_2.

C. Suppose that the government wishes to raise a fixed amount of revenue from the household using separate proportional taxes on L_1 and L_2. Based on the standard three-good analysis, use your answers to part B to derive a condition in terms of the wage rates and utility function parameters for uniform taxation to be optimal.

D. Now, suppose that there are several households, all with the same preferences as those given above but with differing abilities, which may be represented in terms of efficiency units; household i faces wage rates $(w_1 e_1^i, w_2 e_2^i)$, where (e_1, e_2) has the joint distribution $f(e_1, e_2)$. Using this notation, rewrite your expressions for market labor supplies derived in part B. From these new expressions, solve for the household's corresponding supplies of labor in constant efficiency units, $\overline{L}_1^i = L_1^i e_1^i$ and $\overline{L}_2^i = L_2^i e_2^i$.

E. Suppose that the government wishes to use separate proportional taxes on aggregate labor supplies $L_1 = \sum_i \overline{L}_1^i$ and $L_2 = \sum_i \overline{L}_2^i$ to raise a fixed amount of revenue. Unlike in Part C, the optimal policy will now take both efficiency and equity considerations into account. Discuss how equity considerations should depend on the properties of the skill distribution, $f(e_1, e_2)$.

3. A pure consumption tax would treat imputed rent on owner-occupied housing as a consumption flow for purposes of taxation, while the purchases of houses themselves would not be subject to tax. In practice, the taxation of imputed rent is sometimes viewed as impractical, and alternatives have been sought to simulate the taxation of imputed rent.

Consider a two-period life-cycle model in which a household has no initial assets and leaves no bequests. In the first period, the household supplies labor, L, subject to a wage rate w. This labor income is spent on first-period (non-housing) consumption, C_1, the purchase of a house, H, and the purchase of financial assets, A. In the second period, the household sells its financial assets and its house and devotes the proceeds of these sales, $H+A$, plus the interest on the financial assets, rA, to second-period consumption, C_2. The household chooses L, C_1, C_2, and H to maximize a utility function $U(L, C_1, C_2, H)$, where $U_L < 0$ and $U_{C1}, U_{C2}, U_H > 0$.

A. Write down the household's budget constraint and show that it is the same as one in which, rather than buying and selling the house, the household simply rented the house for rH in the second period.

B. Suppose the government wishes to tax imputed rent, raising the effective rental rate on housing from r to $(1+t)r$ in the household budget constraint. Show that a policy of taxing purchases of housing at rate t and providing a rebate at the same rate on sales of housing would achieve this objective.

C. Discuss how the effectiveness of this indirect policy of taxing imputed rent would be affected if the household were endowed with an initial amount of housing, H_0, and purchased housing only to the extent that its desired level of housing, H, exceeded H_0.