1. Consider an economy in which relative producer prices are fixed and a representative household maximizes the following utility function:

\[ U(c_1, c_2, l) = (c_1 - a_1)^{\beta_1} (c_2 - a_2)^{\beta_2} l^{1-\beta_1-\beta_2} \]

(where \( c_1 \) and \( c_2 \) are consumption goods and \( l \) is leisure), subject to the budget constraint:

\[ p_1 c_1 + p_2 c_2 + w l = w \]

A. Derive an explicit expression for the excess burden of taxes on \( c_1, c_2, \) and \( l \) as a function of the original prices of the two goods (\( p_1^0, p_2^0, \) and \( w^0 \)), the distorted prices (\( p_1^1, p_2^1, \) and \( w^1 \)) and a fixed utility level.

B. Compare the values of excess burden based on compensating and equivalent variations.

C. Using the measure derived in part A, show that the deadweight loss is positive for any tax or subsidy on good 2. (Hint: show that marginal deadweight loss has the same sign as \( (p_2^1 - p_2^0) \).

D. Now, suppose that the government wishes to impose proportional taxes on labor income and good-1 consumption. Derive a condition, based only on prices and preference parameters, under which the optimal tax on good 1 will be positive.

2. A pure consumption tax would treat imputed rent on owner-occupied housing as a consumption flow for purposes of taxation, while the purchases of houses themselves – which represent investment – would not be subject to tax. In practice, the direct taxation of imputed rent is sometimes viewed as impractical, and alternatives have been sought to simulate the taxation of imputed rent.

Consider a two-period life-cycle model in which a household has no initial assets and leaves no bequests. In the first period, the household supplies labor, \( L \), subject to a wage rate \( w \). This labor income is spent on first-period (non-housing) consumption, \( C_1 \), the purchase of a house, \( H \), and the purchase of financial assets, \( A \). In the second period, the household sells its financial assets and its house and devotes the proceeds of these sales, \( H + A \), plus the interest on the financial assets, \( rA \), to second-period consumption, \( C_2 \). The household chooses \( L, C_1, C_2, \) and \( H \) to maximize a utility function \( U(L, C_1, C_2, H) \), where \( U_L < 0 \) and \( U_{C1}, U_{C2}, U_H > 0 \).
A. Write down the household’s budget constraint and show that it is the same as one in which, rather than buying and then selling the house, the household simply rents the house for \( rH \) in the second period.

B. Suppose the government wishes to tax imputed rent, raising the effective rental rate on housing from \( r \) to \((1+t)r\) in the household budget constraint. Show that a policy of taxing purchases of housing at rate \( t \) and providing a rebate at the same rate on sales of housing would achieve this objective.

C. Discuss how the effectiveness of this indirect policy of taxing imputed rent would be affected if the household were endowed with an initial amount of wealth in the form of housing, \( H_0 \), and so purchased only \( H - H_0 \) in period 1 before selling \( H \) in period 2.

3. In the Harberger two-sector model, labor bears 100% of an excise tax on sector-\( X \) output if the ratio of capital income to gross expenditures (including the excise tax) is unchanged.

A. For the same assumptions as in the standard Harberger model (e.g., fixed overall supplies of labor and capital, no initial distortions), show that this outcome requires that sector \( X \) be more labor intensive than sector \( Y \). (*Hint:* use the fact that net purchases equals total factor income, so that gross purchases equals total factor income plus tax revenue.)

B. Derive a condition that depends only on factor shares (\( \theta \)), factor allocations (\( \lambda \)) and elasticities of substitution (\( \sigma \)) for labor to bear at least 100% of an excise tax in sector \( X \).

C. Assume that sector \( X \) is more labor intensive than sector \( Y \), so that (from the result in part A) it is possible for labor to bear 100% of an excise tax on sector \( X \). Using the expression you derived in part B, show that, in the limit as goods \( X \) and \( Y \) become perfect substitutes in consumption (i.e., as \( \sigma_D \rightarrow \infty \)), labor must bear at least 100% of the tax.