Economics 250a
Lecture 4, Labor Supply Continued
Outline

1. Simple labor supply models with correction for non-participation (PS #3)
2. Other approaches to kinked budget sets - discretization
3. Dynamic labor supply - introduction and overview

Readings

Thomas A. Mroz. "The Sensitivity of an Empirical Model of Married Women's Hours of Work to Economic and Statistical Assumptions." Econometrica, 55 (4) (July, 1987), pp. 765-799.

Arthur van Soest "Structural models of family labor supply: A discrete choice." Journal of Human Resources 30 (Winter 1995).

Blundell, Macurdy, Meghir (2007)

1. Labor Supply and Corrections for Non-Participation

We briefly discussed "corner solutions" in Lecture 2. We return to this issue using the setup in Mroz's study (which you will be replicating in the next problem set). We are trying to model (say) married women's labor supply, and we are prepared to take their partners' earnings as exogenous. The setup includes a "Stern" labor supply function and a "Mincerian" model for log wages:

$$h_i^* = a_0 + a_1 \log w_i + a_2 y_i + a_3 Z_i + u_{1i} \quad (1a)$$

$$\log w_i = cZ_i + u_{2i} \quad (1b)$$

where Z_i is a vector of background variables (including characteristics of person i, the wife in a 2-person family, as well as the characteristics of her partner and her family). The parameters of interest are a_1 and a_2. Person i is observed working hours h_i^* (or possibly $h_i^* + \epsilon_i$, where ϵ_i, is a measurement error) if

$$h_i^* \geq h_i^{\min}$$

A "pure" reservation wage model assumes $h_i^{\min} = 0$, and (with the Stern labor supply function) gives rise to a Tobit model. A "fixed cost" model assumes

$$h_i^{\min} = b_0 + b_1 Z_i + u_{3i}$$

which allows the lowest-hour job to depend on person-specific characteristics and a stochastic term. Working occurs if

$$a_0 + a_1 \log w_i + a_2 y_i + a_3 Z_i + u_{1i} \geq b_0 + b_1 Z_i + u_{3i}$$

$$\text{or} \quad \xi_i = u_{1i} + a_1 u_{2i} - u_{3i} > -(a_0 - b_0) - a_2 y_i - (a_3 + a_1 c - b_1) Z_i. \quad (2)$$

If we assume that $(u_{1i}, u_{2i}, u_{3i}) \sim N(0, \Sigma)$ we get a very convenient setup. Equation (2) yields a simple probit model for participation with latent normal error ξ_i. With 3 underlying error terms, this error is arbitrarily correlated with the error in the labor supply equation and with the error in the wage equation. (On the other hand, if $u_{3i} = 0$ we have only 2 independent errors). We can estimate both the wage model and the hours equation by 2-step "Heckit", or IV-Heckit.

The "Heckit" approach is a variant of a "control function" approach which uses the fact that if $(z_1, z_2) \sim N(0, \Sigma)$, the distribution of z_1 conditional on the event that $z_2 > c$ has a convenient functional form. Specifically, for joint normals we can always write:

$$z_1 = r_{1,2} z_2 + \nu$$

where $r_{1,2} \equiv cov[z_1, z_2]/var[z_2] = \rho\sigma_1/\sigma_2$ is the "regression coefficient" of z_1 on z_2 and ν is a normal variate with mean 0 that is independent of z_2. Thus

$$\begin{aligned}
E[z_1|z_2 &> c] = r_{1,2} E[z_2|z_2 > c] \\
&= r_{1,2}\sigma_2 E[\frac{z_2}{\sigma_2}|\frac{z_2}{\sigma_2} > \frac{c}{\sigma_2}] \qquad (3)\\
&= \rho\sigma_1 \frac{\phi\left(\frac{c}{\sigma_2}\right)}{1-\Phi\left(\frac{c}{\sigma_2}\right)} = \rho\sigma_1 \frac{\phi\left(\frac{-c}{\sigma_2}\right)}{\Phi\left(\frac{-c}{\sigma_2}\right)} = \rho\sigma_1 \frac{\phi(\Phi^{-1}(p))}{p} \equiv \rho\sigma_1\lambda(p) \\
where \;\; p_1 &= P(z_2 > c) = P(\frac{z_2}{\sigma_2} > \frac{c}{\sigma_2}) = 1 - \Phi\left(\frac{c}{\sigma_2}\right) = \Phi\left(\frac{-c}{\sigma_2}\right)
\end{aligned}$$

and we have used the result that for a standard normal variate ω, $E(\omega|\omega > a) = \phi(\omega)/[1 - \Phi(\omega)]$. Look at this carefully: it says that

$$E[z_1|z_2 > c] = k \cdot \lambda(p)$$

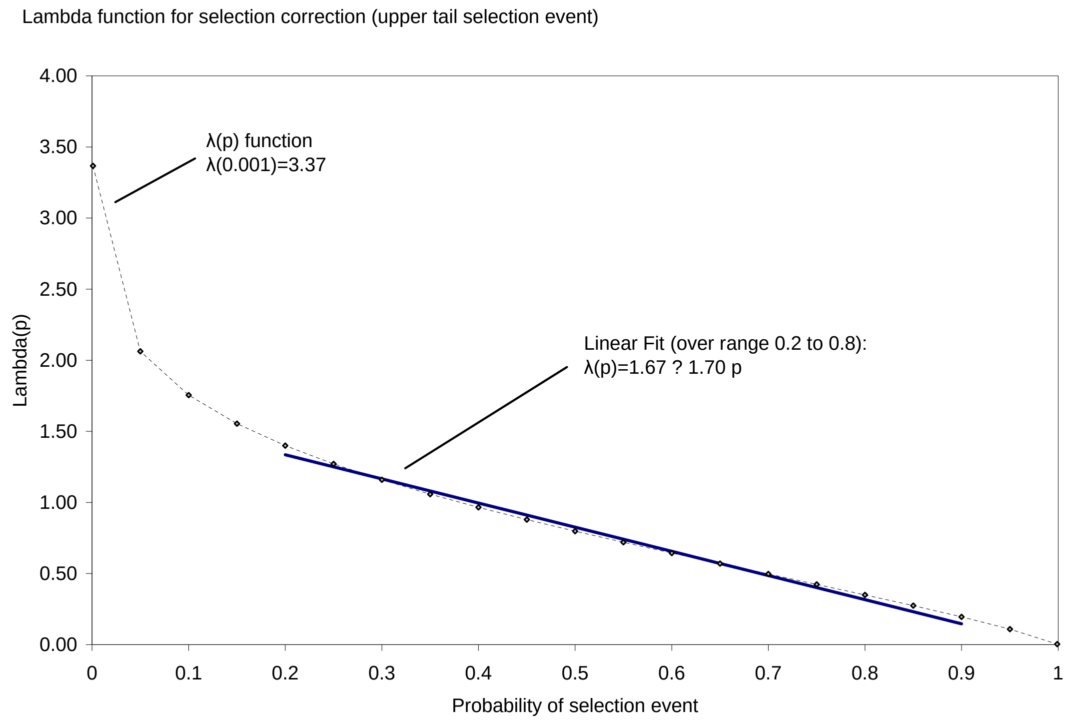
where p is the probability that $z_2 > c$, k is a constant that has the same sign as ρ, the correlation of $z_1$and z_2, and $\lambda(p) = \frac{\phi(\Phi^{-1}(p))}{p}$is a known function of p (see the plot below). In our application $z_2 > c$ is the condition for working (with probability p) and we will have 2 $z_1's$: hours and wages, neither of which is observed for non-workers.

In a 2-stage Heckit you first estimate the parameters of a reduced form probit model that determines whether the outcome (or outcomes) of interest are observed. You then take the parameters of this model, use them to predict p for each observation in the "observed" data set, and form $\lambda(p)$. In the observed sample

$$\begin{aligned}
h_i &= a_0 + a_1 \log w_i + a_2 y_i + a_3 Z_i + E[u_{1i}|\xi_i > -(a_0 - b_0) - a_2 y_i - (a_3 + a_1 c - b_1) Z_i] + \nu_{1i} \\
&= a_0 + a_1 \log w_i + a_2 y_i + a_3 Z_i + k_1 \lambda(p) + \nu_{1i}
\end{aligned}$$

and

$$\begin{aligned}
\log w_i &= cZ_i + E[u_{2i}|\xi_i > -(a_0 - b_0) - a_2 y_i - (a_3 + a_1 c - b_1) Z_i] + \nu_{2i} \\
&= cZ_i + k_2 \lambda(p) + \nu_{2i}
\end{aligned}$$



If we are worried that $\log w_i$ or y_i are endogenous (or measured with a lot of error) then we have to apply "IV-Heckit": step 1 = probit for participation, step 2 = form $\lambda(p)$,step 3 estimate the Heckit labor supply model using $\lambda(p)$ and whatever instruments are selected for wages and/or non-labor income. Another way to proceed would be to substitute $\log w_i = cZ_i + u_{2i}$ into the hours equation, yielding

$$h_i^* = a_0 + (a_1 c + a_3)Z_i + a_2 y_i + u_{1i} + a_1 u_{2i}]$$

which is only observed for participants. So we get a selection-corrected reduced form hours model

$$h_i = a_0 + (a_1 c + a_3)Z_i + a_2 y_i + k_3 \lambda(p) + \nu_{3i}$$

We can estimate the parameters c using a selection-corrected wage model, and the parameters $(a_1 c + a_3)$ and a_2 using a selection-corrected hours model. We can then do method of moments to infer a_1.

An even more complex situation arises when we want to instrument y_i (either because of measurement error, or a concern that some of the variables that drive y_i also affect hours choices. This would lead to IV-Heckit with 2 endogenous variables

Questions for discussion:

1) We clearly need instruments for the wage (given division bias). What are plausible candidates?

2) We also need variables that move $\lambda(p)$ around – so variables that determine the probability of participation but do not directly affect wages and hours. Thoughts?

2. Other approaches to kinked budget sets - discretization

Kinked budget sets and non-participation can be handled in a "unified" way by thinking of the budget set as consisting of discrete points. For example, we could discretize hours choices into 1 hour "bins", or 5 hour bins, or even 3 choices: h=0, h=20, h=40 (non-work, part-time, and full-time). This is arguably the 'state of the art' for static labor supply.

The simplest implementation is to pick a parametric utility function $u(x, h; \theta, z, \epsilon)$, where θ represent parameters, z represent observable characteristics, and ϵ represents unobserved heterogeneity. Then assume some simple way of classifying observed choices into the discrete options $\{h_0 = 0, h_1, h_2, ...h_J\}$, and make a set of assumptions that allow you to associate an earnings amount $\{e_1(w), e_2(w), ...e_J(w)\}$ with each choice for a worker with wage w. (The earnings amounts $e_j(w)$ can incorporate taxes and transfers, a premium or discount for part-time work, a fixed cost of working, etc). A worker with wage w and heterogeneity (z, ϵ) evaluates $u(y + e_j(w),\ h_j; \theta, z, \epsilon)$ for each option and chooses the highest utility. In practice there are a lot of delicate issues: (1) how to deal with measurement error in hours; (2) how to parameterize the earnings opportunity set; (3) how to put random taste variation in the ϵ vector. A starting point for this line of work is usually a multinomial logit model

$$u(y + e_j(w), h_j; \theta, z, \epsilon) = v(y + e_j(w), h_j; \theta, z) + \epsilon_j$$

where ϵ_j are extreme value type 1 errors, and v is some convenient functional form. If the wage w was known for each person this would lead to a simple MNL choice model

$$\Pr(choice\ j \mid w, z) = \frac{\exp(v(y + e_j(w), h_j; \theta, z))}{\sum_k v(y + e_k(w), h_k; \theta, z)}$$

Since the wage is not observed for non-workers (and may be measured with error) a standard approach is to assume a d.g.p. for wages that implies a density $f(w|z)$, and treat w as an endogenous outcome. Then the likelihood for the observed workers is of the form:

$$\Pr(choice\ j, w \mid z) = \Pr(choice\ j \mid w, z) \times f(w \mid z).$$

The likelihood for non-workers is

$$\Pr(nonwork \mid z) = \int \Pr(choice\ j = 0 \mid w, z) \times f(w \mid z) dw$$

An example of this approach is van Soest (1995). A more recent variant is:

Arthur van Soest, Marcel Das and Xiaodong Gong, "A Structural Labour Supply Model with Flexible Preferences". Journal of Econometrics 107 (2002), pp. 345-374.

3. Intertemporal Labor Supply

Intertemporal labor supply (ils) models are used to analyze responses to variation over time in wages or income. There are three main "dimensions" of intertemporal labor supply: (1) the pure lifecycle dimension. Wages have a hump-shaped pattern over the lifecycle. Do people respond to this variation? (2) the "macro" dimension. Hours of work vary over the business cycle. Can we interpret this as an endogenous response to wages? (3) the "idiosyncratic" dimension. A person may have temporarily higher wages in some period. How does he/she respond?

Denote periods (e.g., years) by "t". Consumption in period t is c_t, hours of work are h_t, the wage is w_t. Individuals have an additively separable intertemporal utility function

$$u(c_t, h_t; a_t) + \beta u(c_{t+1}, h_{t+1}; a_{t+1}) + \beta^2 u(c_{t+2}, h_{t+2}; a_{t+2}) + ...$$

where a_t is a "preference shock" in period t. The intertemporal budget contraint is

$$A_{t+1} = (1 + r_t)(A_t + y_t + w_t h_t - c_t)$$

where A_t represent real assets in period t, and r_t is the real interest rate from t to $t + 1$. The Bellman equation is

$$V_t(A_t) = \max_{c_t, h_t} u(c_t, h_t; a_t) + \beta E_t[V_{t+1}((1 + r_t)(A_t + y_t + w_t h_t - c_t))]$$

The f.o.c. are

$$\begin{aligned} u_c(c_t, h_t; a_t) - \beta(1 + r_t)E_t[V'_{t+1}(.)] &= 0 \\ u_h(c_t, h_t; a_t) + \beta w_t(1 + r_t)E_t[V'_{t+1}(.)] &= 0 \end{aligned}$$

Define

$$\lambda_t \equiv V'_t(A_t) = \beta(1 + r_t)E_t[V'_{t+1}(.)] = \beta(1 + r_t)E_t[\lambda_{t+1}],$$

where the second equality is from the envelope theorem and the third just uses the definition of λ_{t+1}. We can rewrite the f.o.c. as

$$\begin{aligned} u_c(c_t, h_t; a_t) &= \lambda_t \\ u_h(c_t, h_t; a_t) &= -w_t \lambda_t \end{aligned}$$

implying a "tangency condition" (or "within-period efficiency condition")

$$\frac{-u_h(c_t, h_t; a_t)}{u_c(c_t, h_t; a_t)} = w_t$$

Note that the budget constraint within the period is

$$c_t = w_t h_t + y_t - [\frac{A_{t+1}}{1 + r_t} - A_t]$$

So the within-period choices are the same as a person with the same utility function would make in a static problem with non-labor income

$$\begin{aligned} & y_t - S_t, \qquad \text{where} \\ S_t &= \frac{A_{t+1}}{1 + r_t} - A_t \end{aligned}$$

The within-period f.o.c. define "Frisch" demands

$$\begin{aligned} c_t &= c^F(w_t, \lambda_t, a_t) \\ h_t &= h^F(w_t, \lambda_t, a_t) \end{aligned}$$

These are also called the "lambda-constant" or "intertemporal" consumption and hours choices. There are 4 elasticities

$$\frac{\partial \log h^F}{\partial \log w}, \frac{\partial \log h^F}{\partial \log \lambda}, \frac{\partial \log c^F}{\partial \log w}, \frac{\partial \log c^F}{\partial \log \lambda}.$$

The response of hours to wages holding constant λ is called "the intertemporal substitution elasticity". Recall there are also compensated and uncompensated responses

$$\frac{\partial \log h^c(w, u)}{\partial \log w}, \frac{\partial \log h(w, y)}{\partial \log w}$$

We showed in the first lecture that $\frac{\partial \log h^F}{\partial \log w} \geq \frac{\partial \log h^c(w,u)}{\partial \log w} \geq 0$. Looking at the f.o.c. we get

$$\begin{pmatrix} dc \\ dh \end{pmatrix} = \begin{bmatrix} U_{cc} & U_{ch} \\ U_{hc} & U_{hh} \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ -w & -\lambda \end{bmatrix}$$

So

$$\frac{\partial h^F}{\partial w} = \frac{1}{U_{cc}U_{hh} - U_{ch}^2} \begin{vmatrix} U_{cc} & 0 \\ U_{hc} & -\lambda \end{vmatrix} = \frac{-\lambda U_{cc}}{\Delta}$$

The other derivatives are

$$\begin{aligned} \frac{\partial h^F}{\partial \lambda} &= \frac{-wU_{cc} - U_{hc}}{\Delta} \\ \frac{\partial c^F}{\partial \lambda} &= \frac{wU_{ch} + U_{hh}}{\Delta} \\ \frac{\partial c^F}{\partial w} &= \frac{\lambda U_{ch}}{\Delta} \end{aligned}$$

Using these relations we can show that

$$\frac{\partial \log h^F}{\partial \log w} = \frac{\partial \log h^F}{\partial \log \lambda} + \frac{c}{wh} \frac{\partial \log c^F}{\partial \log w}$$

So in the separable case ($U_{ch} = 0$), we get that the $\frac{\partial \log h^F}{\partial \log w} = \frac{\partial \log h^F}{\partial \log \lambda}$. More generally if $c \approx wh$ this shows that hours have to vary more than consumption in response to "expected" wage variation. So an agent should increase savings when the wage is high.

EXERCISE: Consider an agent with a given per-period utility function $u(c,h)$ who solves a 1-period optimization:

$$\max_{c,h} u(c,h) \quad \text{s.t.} \quad c = wh + y$$

Using Cramer's rule, find $\frac{\partial h}{\partial w}, \frac{\partial h}{\partial y}$. Show that

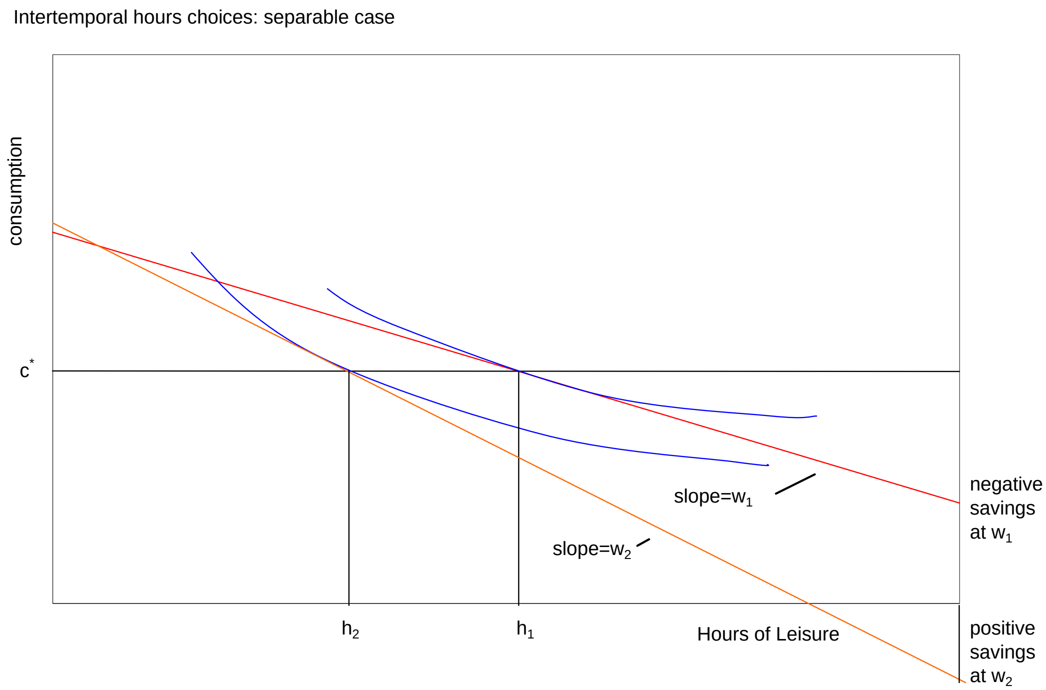
$$\frac{\partial h^c}{\partial w} = \frac{-\lambda}{w^2 U_{cc} + 2wU_{ch} + U_{hh}}$$

and that

$$\frac{1}{\frac{\partial h^c}{\partial w}} - \frac{1}{\frac{\partial h^f}{\partial w}} \geq 0.$$

What is the intertemporal substitution elasticity? Start with the case where $U(c,h) = \phi(c) - \psi(h)$. Then $u_c = \lambda_t$ means that $c_t = c^* = const.$ (holding constant a_t). This was Friedman's assumption in the PIH model. In this case, as wages vary along an expected profile (so $\lambda_t = E_{t-1}[\lambda_t]$) the agent finds an hours choice such that

$$\begin{aligned} \frac{-u_h(c^*, h_t)}{u_c(c^*, h_t)} &= w_t \\ i.e.\ \psi'(h_t) &= \lambda_t w_t. \end{aligned}$$



This is illustrated here:

Note that utility $U(c,h) = \phi(c^*) - \psi(h_t)$ is *lower* in high wage-periods, but the person also gains savings,which have marginal value $\lambda_t = \lambda = \phi'(c^*)$: so "flow utility" is

$$\phi(c^*) - \psi(h_t) + \lambda(w_t h_t - c^*).$$

In the non-separable case we have a "locus" of points (c,h) s.t. $U_c(c,h) = \lambda$. As wages vary along an expected trajectory the agent finds a point on this locus with $\frac{-u_h(c,h)}{u_c(c,h)} = w$. Note that (ignoring preference changes over time):

$$U_c(c^F(w_t,\lambda), h^F(w_t,\lambda)) = \lambda$$

implying that

$$\frac{\partial c^F(w_t,\lambda)}{\partial w} = -\frac{U_{ch}}{U_{cc}} \frac{\partial h^F(w_t,\lambda)}{\partial w}$$

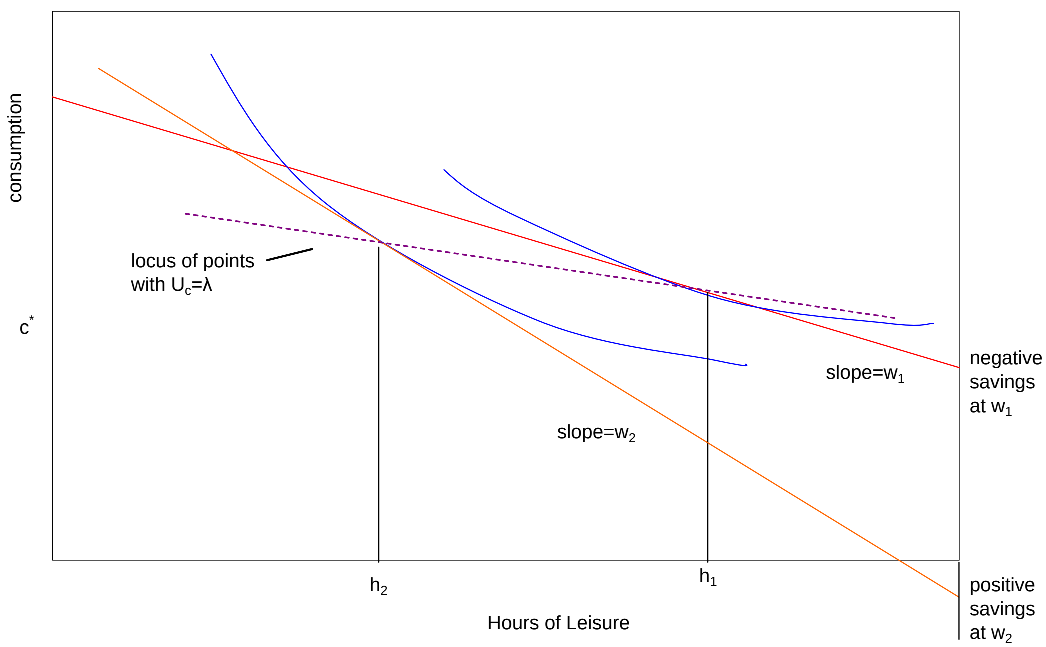
and since $\frac{\partial h^F(w_t,\lambda)}{\partial w} > 0$, and $U_{cc} < 0$, the agent will consume more or less in high-wage periods depending on the sign of U_{ch}. It is commonly asserted that $U_{ch} > 0$. In this case the locus is negatively sloped, as shown in the next picture.

Dealing with Uncertainty

What happens if there are unexpected changes? Let's log-linearize the f.o.c.:

$$\log h_t = a_t' + \eta \log w_t + \delta \log \lambda_t$$

Intertemporal hours choices: non-separable case



And in the previous period

$$\log h_{t-1} = a'_{t-1} + \eta \log w_{t-1} + \delta \log \lambda_{t-1}$$

Differencing we get

$$\Delta \log h_t = \log h_t - \log h_{t-1} = \Delta a'_t + \eta \Delta \log w_t + \delta(\log \lambda_t - \log \lambda_{t-1})$$

But from the f.o.c recall

$$\lambda_{t-1} = \beta(1+r)E_{t-1}[\lambda_t]$$

so

$$\log \lambda_{t-1} = \log[\beta(1+r)] + \log E_{t-1}[\lambda_t]$$

Now let

$$\log E_{t-1}[\lambda_t] = E_{t-1} \log[\lambda_t] + \phi_t \quad \text{(see note ♦)}$$

and define the "innovation" in the log marginal utility of income as ξ_t where:

$$\log \lambda_t = E_{t-1} \log[\lambda_t] + \xi_t$$

Combining all these terms we get

$$\begin{aligned} \log \lambda_{t-1} &= \log[\beta(1+r)] + \log E_{t-1}[\lambda_t] \\ &= \log[\beta(1+r)] + E_{t-1} \log[\lambda_t] + \phi_t \\ &= \log[\beta(1+r)] + \log \lambda_t - \xi_t + \phi_t \end{aligned}$$

or

$$\log \lambda_t = \log \lambda_{t-1} - \log[\beta(1+r)] - \phi_t + \xi_t$$

So

$$\Delta \log h_t == \Delta a_t^{'} + \eta \Delta \log w_t + \delta \xi_t - \delta(\phi_t + \log[\beta(1+r)] \quad (4)$$

This says that the change in hours depends on:

the change in tastes $\Delta a_t^{'}$
the change in wages (with an elasticity η)
the innovation in the log MU of income (with an elasticity $\delta \geq 0$)
two terms which are constants

♦Aside

what is the relationship of $E[\log x]$ and $\log E[x]$? One easy case is when x is log-normally distributed: $\log x \sim N(\mu, \sigma^2)$. (Wages are approximately log-normal so this is often a useful case for labor economics). In the log-normal case:

$$E[x] = \exp(\mu + \frac{1}{2}\sigma^2)$$

so

$$\log E[x] = \mu + \frac{1}{2}\sigma^2 = E[\log x] + \frac{1}{2} var[\log x]$$

In the labor supply setting, then, if λ_t was log-normal we would have that

$$\phi_t = \frac{1}{2} var_{t-1}[\log \lambda_t]$$

which could well vary over time or across people.

Empirical Wage Processes

In the 1970s economists realized that in a lifecycle labor supply model, the responsiveness of an individual to an observed wage change will depend on two things: (1) how much of the change was anticipated (2) how much of the change is expected to persist. The labor supply elasticity η only pertains to fully anticipated wage changes, which leave λ_t unchanged and lead to changes in hours $\eta \Delta \log w$. *Unanticipated* wage changes have "wealth effects." An unexpected rise in w_t will lower $\log \lambda_t$, especially to the extent it is expected to persist.

A convenient "approximate" intertemporal labor supply model that was first introduced by Milton Friedman, and later used in a famous paper by Lucas and Rapping, is

$$\log h_t = \eta[\log w_t - (1-\beta) \sum_{j=0} \beta^j E_t \log w_{t+j}]$$

In this model, a permanent increase in wages has no effect, while a temporary wage increase has an effect of $+\eta$. You can check that this is equivalent to assuming that within-period preferences are $u(c,h) = \phi(c) - \psi(h)$, with $\psi(h) = (1+\frac{1}{\eta})^{-1} h^{1+\frac{1}{\eta}}$, and

$$\log \lambda_t = -(1-\beta) \sum_{j=0} \beta^j E_t \log w_{t+j}$$

which is hard to rationalize.

Given the importance of distinguishing between persistent and transitory wage changes, a lot of research has asked how real wages evolve over the life cycle (e.g., Abowd and Card, 1989). One model for individual wages is:

$$\begin{aligned} \log w_{it} &= \omega_i + u_{it} \quad , \\ u_{it} &= u_{it-1} + \zeta_{it} \end{aligned}$$

where the $\zeta_{it}'s$ are uncorrelated over time. This is a "pure random walk" model, in which $E[\log w_{it+j} | \log w_{it}] = \log w_{it}$. A more general model is

$$\begin{aligned} \log w_{it} &= \omega_i + x_{it}\beta_t + u_{it} + e_{it} \\ u_{it} &= \alpha u_{it-1} + \zeta_{it} \quad , \end{aligned}$$

where e_{it} and ζ_{it} are serially uncorrelated and uncorrelated with each other. This model includes a fixed component ω_i, a component attributable to observables x_{it}, an AR(1) component u_{it}, and a "purely transitory" component e_{it} perhaps due to measurement error. A third class of models includes a person-specific growth component:

$$\begin{aligned} \log w_{it} &= \omega_i + \rho_i t + x_{it}\beta_t + u_{it} + e_{it} \\ u_{it} &= \alpha u_{it-1} + \zeta_{it} \quad , \end{aligned}$$

There is some controversy in the literature over (1) whether the AR(1)-like component has a unit root or less than a unit root; (2) whether there are person-specfic growth terms like ρ_i in wages.