Economics 250a Lecture 11
Search Theory 2

Outline
a) search intensity - a very simple model
b) The Burdett-Mortensen equilibrium wage-posting model (from Manning)
c) Brief mention - Christensen et al (2005)
d) DMP search and matching model

References
Kenneth Burdett and Dale Mortensen (1998) "Wage Differentials Employer Size and Unemployment" *International Econ Review* 39, pp. 257-273.

Alan Manning (2003). *Monopsony in Motion.* Princeton: PU Press.

Bent Christensen, Rasmus Lentz, Dale Mortensen and George Neumann (2005). "On the Job Search and the Wage Distribution". JOLE 23 (January 2005), pp. 31-58

Christopher Pissarides Equilibrium Unemployment Theory. Cambridge MA: MIT Press, 1990 (Second edition 2000).

a) A Very Basic Model with Search Intensity

The following model is a simplification of the one presented in Card-Chetty-Weber. It assumes that a given worker will receive a wage w if employed (so there is no uncertainty about wages offers). With no risk of job destruction the value function for employment at wage w is:

$$
\begin{aligned}
U(w) &= w + \beta U(w) \\
\Rightarrow U(w) &= \frac{w}{1-\beta}
\end{aligned}
$$

A worker who is unemployed chooses search intensity s which is just the probability of finding a job. The value function for unemployment at benefit b for a worker who will get a wage w when she/he finds a job is:

$$
V(b,w) = max_s\{b + \beta[sU(w) + (1-s)V(b,w)] - \psi(s)\}
$$

where $\psi(s)$ is the (convex) cost of job search effort. The optimal level of search effort s^* solves the FOC:

$$
\psi'(s^*) = \beta[U(w) - V(b,w)]
$$

And

$$
V(b,w) = \frac{b + \beta s^* U(w) - \psi(s^*)}{1 - \beta(1 - s^*)}
$$

Notice that we can get the derivatives of $V(b, w)$ directly from this expression, ignoring the endogeneity of s^*, from the envelope theorem. So we have:

$$U_w = \frac{1}{1+\beta}$$

$$V_w = \frac{\beta s^*}{1-\beta+\beta s^*} U_w < U_w$$

$$V_b = \frac{1}{1-\beta(1-s^*)}$$

So

$$\psi''(s^*)\frac{\partial s^*}{\partial b} = -\beta V_b$$

$$\psi''(s^*)\frac{\partial s^*}{\partial w} = \beta[U_w - V_w]$$

Since $\psi''(s^*) > 0$ we have that s^* decreases with b and increases with w. You can port this same basic structure to a "job ladder" model of the type explored in Lecture 10, and get similar basic results.

b) Manning's simplification of Burdett-Mortensen's wage posting model

The BM model is an equilibrium search model in which atomistic employers each choose (or post) a wage, and employees search on the job. The model generates an equilibrium distribution of wage offers, F, an associated distribution of accepted wages G, an equilibrium rate of unemployment (or strictly speaking non-employment) u. Firms with higher wages are larger and have a lower attrition rate (because fewer workers are bid away). Firms with lower wages are smaller and have higher attrition. All wage choices yield the same profit. The model is an extension of a famous paper by Burdett (1978) which first proposed the idea of job ladders – on the job search by employed who are looking for a wage at a higher wage firm.

Economists differ in their enthusiasm for the BM model and related wage posting models. Some argue that it does not make sense for firms to refuse to re-negotiate with workers who find a better-paying job (i.e., a firm that posts a single wage is committing to *no offer-matching*). The empirical importance of offer matching is an open subject for research.

As a prologue, some evidence of the importance of wage posting comes from a survey by Hall and Krueger (NBER #16033, May 2010): they find that only about 30% of workers report there was some bargaining in setting the wage for their current job. The rate is especially low for blue collar workers (5%) but much higher for knowledge workers (86%).

We will follow Manning's exposition (MM, chapter 2). This derivation underlines the "statistical mechanics" of BM - behavior is summarized by the strategies of unemployed and employed searchers.

Key notation:

-M_w workers, all equally productive, each has non-work option b

-M_f firms, each has constant productivity per worker p.

- $M = M_f/M_w$

- each firm offers wage w for all its workers

-$F(w)$ = distribution of wages across firms (to be determined)

-employed and nonemployed workers receive offers randomly (from F) at rate λ. A GE variant could make λ endogenous.

-employed workers leave for nonemployment at rate δ

Steady State Behaviors

a) As in our basic model with on the job seach, non-employed workers accept any job offer paying more than b. Employed workers accept any job paying more than their current wage.

b) Steady-state profits

$$\pi(w; F) = (p - w)N(w; F)$$

where $N(w; F)$ is the steady state level of employment at a firm paying w given F. We are going to assume π is the same for all firms, which means small firms have lower wages and make higher profit per worker.

c) Balancing flows: firm w has separation rate $s(w; F)$ and recruiting flow $R(w; F)$. So in steady state

$$s(w; F)N(w; F) = R(w; F).$$

d) (no spikes). If $0 < \lambda/\delta < \infty$, $F(w)$ has no spikes (atoms). Why? If there is a spike at some wage $w_0 < p$ then another firm could guarantee a higher level of profit by offering a wage $w_0 + \epsilon$. This has only slightly lower profit per worker but a discretely higher recruiting rate, contradicting the equal profit condition. (If $w_0 = p$ then a firm could make strictly positive profit by lowering its wage).

e) Separation rate for firm paying w is:

$$s(w; F) = \delta + \lambda(1 - F(w)).$$

f) Steady state non-employment. The outflow from non-emp is $\lambda u M_w$, the inflow is $\delta(1 - u)M_w$, balance gives:

$$u = \frac{\delta}{\delta + \lambda} \Rightarrow \frac{u}{1 - u} = \frac{\delta}{\lambda}$$

This relationship holds in any steady state "flow" model and is useful to remember.

g) Distribution function of wages across workers (i.e., fraction of workers earning less than some wage w is $G(w)$, which is not the same as $F(w)$, because firm size varies with w. In fact:

$$G(w; F) = \frac{\delta F(w)}{\delta + \lambda(1 - F(w))} = \frac{F(w)}{1 + \frac{\lambda}{\delta}(1 - F(w))} < F(w) \quad (*)$$

To prove this: consider the set of jobs with $\tilde{w} \leq w$. The size of the pool of workers in these jobs is $(1 - u)G(w)M_w$ (by definition of $G(w)$).

- net entry to this set is from the pool of unemployed. The inflow is $uM_w\lambda F(w)$ since a share $\lambda F(w)$ of the unemployed get offers from b to w.

- net exit from this set has two parts: job destruction flow$=\delta(1-u)G(w)M_w$ and exit to higher wage jobs$= \lambda(1-u)G(w)(1 - F(w))M_w$.

- equating inflow and outflow we get:

$$u\lambda F(w) = \delta(1-u)G(w)+\lambda(1-u)G(w)(1-F(w)) = (1-u)G(w)(\delta+\lambda(1-F(w))$$

Simplifying yields $(*)$. Note that $(*)$ implies as $\lambda \to 0$ $G \to F$.

h) Flow of recruits to the firm. A firm that pays w gets new workers from 2 sources. It gets a share of the non-employed who received an offer that does not depend on its wage: the flow rate from the non-employed pool is

$$\lambda u \frac{M_w}{M_f} = \frac{\lambda u}{M} \quad \text{where } M = \frac{M_f}{M_w}.$$

It also gets a share of all those who are employed at a wage less than w. The flow rate from this group is

$$(1 - u)\lambda G(w; F)\frac{M_w}{M_f} = \frac{(1 - u)\lambda G(w; F)}{M}$$

Thus

$$\begin{aligned} R(w; F) &= \frac{\lambda}{M}(u + (1 - u)G(w; F)) \\ &= \frac{\lambda}{M}\left(\frac{\delta}{\delta + \lambda} + \frac{\lambda}{\delta + \lambda}\frac{\delta F(w)}{\delta + \lambda(1 - F(w))}\right) \\ &= \frac{\lambda\delta}{M}\left(\frac{1}{\delta + \lambda(1 - F(w))}\right). \end{aligned}$$

Finally, using $sN = R$ and substituting we get

$$N(w; F) = \frac{R(w; F)}{s(w; F)} = \frac{\lambda\delta}{M[\delta + \lambda(1 - F(w))]^2}$$

Thus a firm that pays a higher wage will have more employees. Note that in this model:

$$R(w; F) = \frac{\lambda\delta}{M}\frac{1}{s(w; F)}$$

which implies that the elasticity of the recruiting rate with respect to the wage is just the negative of the elasticity of the separation rate with respect to the wage. Manning (p. 97) presents another derivation of this relation, which has to hold quite generally.

We are finally ready for the last step: substituting N into the equation for profits we get

$$
\begin{aligned}
\pi(w; F) &= (p - w)N(w; F) \\
&= \frac{\lambda\delta(p - w)}{M[\delta + \lambda(1 - F(w))]^2}
\end{aligned}
$$

In equilibrium all offered wages give the same level of profit, and no other possible wages yield higher profit. To solve for the equilibrium level of profit, BM (and Manning) show that the lowest wage offered in equilibrium is b. The basic point of the proof is that if the lowest equilibrium wage offered is above b, then a firm at this position could lower its wage and get the same flow of recruits (all of whom are coming from non-employment) and have a lower wage - so there would be a contradiction. Using this fact, we get

$$
\pi(w; F) = \pi(b; F) = \frac{\lambda\delta(p - b)}{M[\delta + \lambda]^2} = \frac{u(1 - u)(p - b)}{M}
$$

Finally, then, once can solve for $F(w)$. The results are:

$$
b \leq w \leq p - \left(\frac{\delta}{\delta + \lambda}\right)^2 (p - b)
$$

$$
\begin{aligned}
F(w) &= \frac{\delta + \lambda}{\lambda}\left(1 - \sqrt{\frac{p - w}{p - b}}\right) \\
G(w) &= \frac{\delta}{\lambda}\left(\sqrt{\frac{p - b}{p - w}} - 1\right) \\
E[w] &= \frac{\delta}{\delta + \lambda}b + \frac{\lambda}{\delta + \lambda}p
\end{aligned}
$$

Notice that as $\lambda \to \infty$, $E[w] \to p$ (search frictions disappear), whereas as $\lambda \to 0, E[w] \to b$ which is the so-called "Diamond paradox". A problem for the model is that the implied shape of the wage distribution is very unrealistic.

c) Christensen et al. (2005)

CLMN use a variant of the BM setup, and estimate some of the underlying parameters using a relatively simple database from Denmark that contains wages earned by workers at each firm in 12-month period, the "exit rate" of workers from each firm, and the number of people hired at the firm who were previously not working. The twist is that now each worker chooses a search

intensity. In general, search intensity will be decreasing with the current wage. They do not try to "explain" how higher and lower wage firms can co-exist. Nevertheless the paper has some interesting ideas for thinking about mapping a BM type world to real data.

Some basic features:

a) They use the wage distribution for people who were hired from non-employment to estimate the distribution $F(w)$.

b) they use the exit hazard rate of workers at firms with different wages to infer the (normalized) seach intensity $\lambda(w)$. As in the baseline BM model, net exit from a firm that pays a wage w has two components: job destruction which is independent of wages at rate δ; and exit to higher wage jobs, at rate $\lambda(w)(1 - F(w))$. Thus the exit hazard is

$$d(w) = \delta + \lambda(w)(1 - F(w))$$

The "hard work" in the paper is to get an expression for $\lambda(w)$, which depends on the optimal choice of search intensity by a worker who is receiving a wage w and gets offers from a distribution $F(w)$. This is derived as follows.

Let $W(w)$ denote the value function for a job paying wage w, and let U denote the value of unemployment. The continuous time Bellman equation is:

$$rW(w) = max_s w - c(s) + \lambda_0 s \int_x (max[W(x), W(w)] - W(w)) \, dF(x) + \delta(U - W(w))$$

Note that the net arrival rate of offers is $\lambda_0 s$ where λ_0 is some scale factor. Re-organizing this can be written as

$$W(w) = max_s \left(\frac{w - c(s) + \delta U + \lambda_0 s \int_x max[W(x), W(w)] dF(x)}{r + \delta + \lambda_0 s} \right)$$

Next, using the envelope theorem we can show (as in the simple model at the start of the lecture) that:

$$W'(w) = \frac{1}{r + \delta + \lambda_0 s(w)(1 - F(w))}$$

Finally, the FOC for optimal search (using integration by parts) is:

$$
\begin{aligned}
c'(s) &= \lambda_0 \int_w^\infty [W(x) - W(w)] dF(x) \\
&= \lambda_0 \int_w^\infty [W'(x)(1 - F(x)) dx \\
&= \lambda_0 \int_w^\infty \frac{(1 - F(x)) dx}{r + \delta + \lambda_0 s(w)(1 - F(w))}
\end{aligned}
$$

CLMN assume $c(s)$ is a convex function with constant elasticity, so $c'(s) = c_0 s^{1/\gamma}$ so this solves out to

$$\lambda(w) \equiv \lambda_0 s(w) = K \left[\int_w^\infty \frac{(1 - F(x))dx}{r + \delta + \lambda(w)(1 - F(w))} \right]^\gamma$$

(where K depends on c_0 and λ_0). This is a functional equation for $\lambda(w)$.

So CLMN actually estimate the exit hazard $d(w)$ and given $F(w)$ solve for (δ, K, γ) assuming r is known.

d) DMP

This is a very important class of models that essentially endogenize the arrival rate of offers. The new constuct is the idea of "vacancies", which represent the employer side of the market.

The economy is in steady state. There are L workers, uL are unemployed, where u is the unemployment rate. There are also vL job vacancies. There is a match function $M(uL, vL)$ that gives the rate of "job match creation". It is assumed that M has CRS so we can write: $M(uL, vL) = vM(\frac{u}{v}, 1)$. Define $\theta = v/u$ as "tightness" and $q = M/vL$ as the vacancy filling rate, so

$$\begin{aligned} q &= M(uL, vL)/vL \\ &= M(\frac{u}{v}, 1) \\ &= q(\theta) \end{aligned}$$

Notice that

$$q(\theta) = M(\frac{1}{\theta}, 1) \Rightarrow q'(\theta) = \frac{-1}{\theta^2} M_1 < 0,$$

so when there are more vacancies per searcher, the rate of v-filling will fall. On the other hand the rate that $U's$ find jobs is

$$M/uL = \theta q(\theta) = \theta M(\frac{1}{\theta}, 1) = M(1, \theta)$$

which is increasing in θ. This is the equivalent of λ – the arrival rate of offers.

Job creation and destruction

The total rate of job creation is $\theta q(\theta) \times uL$. The rate of job destruction is $\delta(1 - u)L$. As usual the steady state rate of unemployment is

$$u = \frac{\delta}{\delta + \theta q(\theta)} = \frac{\delta}{\delta + M(1, \frac{v}{u})}$$

which is a relation between u and v – this is downward sloping and depends on the match function and the rate of job destruction. For example, set $M = U^{.5}V^{.5}$ then we get

$$v = \frac{\delta^2(1 - u)^2}{u}$$

which is plotted at the end of the lecture. This is called the Beveridge Curve.

Next we need to get a model for vacancy creation. Each firm decides to post a vacancy or not. There is a flow cost c of a vacancy. Letting V represent the value of an unfilled vacancy and J the value of a filled vacancy we have:

$$rV = -c + q(\theta)(J - V)$$

If we assume jobs are created until $V = 0$ we get an equilibrium condition

$$J = c/q(\theta)$$

so the value of a job equals the flow cost c times $1/q$ which is the expected time to fill the job - so $c/q(\theta)$ is expected cost to fill a job.

Now if output of a job is p and the wage is w and the destruction rate is δ we also have a Bellman equation:

$$
\begin{aligned}
rJ &= p - w - \delta J \\
\Rightarrow J &= \frac{p - w}{r + \delta}
\end{aligned}
$$

so in equilibrium we must have

$$p - w = (r + \delta)c/q(\theta)$$

or

$$w = p - (r + \delta)c/q(\theta)$$

Wages have to be less than p by an amount that is the amortized cost of creating the job. This is the equilibrium inverse labor demand equation.

Finally we have to determine how wages are set, once a match is formed. (Note that wages do not affect the job creation rate). The standard way is to figure out the value of a newly created job to workers, and assume that the wage is determined to split the net value to the worker PLUS the firm. Defining U as the value of unemployment and $W(w)$ as the value of a job we get

$$
\begin{aligned}
rU &= b + \theta q(\theta)(W(w) - U) \\
rW(w) &= w + \delta(U - W(w))
\end{aligned}
$$

(so we are ignoring on the job search). Note that the second eq. implies

$$W(w) = \frac{w}{r + \delta} + \frac{\delta}{r + \delta}U$$

and using this with the expression for rU we get

$$rU = \frac{b(r + \delta) + \theta q(\theta)w}{r + \delta + \theta q(\theta)} \quad (*).$$

which in equilibrium depends on the wage that will be recieved by workers. Holding constant U the lowest wage the worker will accept, w^*, has $W(w^*) = U$ which implies

$$rU = w^*$$

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And then
$$W(w) - U = \frac{w}{r+\delta} - \frac{r}{r+\delta}U = \frac{w - w^*}{r+\delta}$$

Now when the worker and the firm "match" the total surplus created (holding constant U) is

$$
\begin{aligned}
S &= W(w) - U + J \\
&= \frac{w - w^*}{r+\delta} + \frac{p - w}{r+\delta} \\
&= \frac{p - w^*}{r+\delta}
\end{aligned}
$$

which does not depend on w. The standard assumption is wages solve the "Nash" problem:

$$max_w (\frac{w - w^*}{r+\delta})^\beta (\frac{p - w}{r+\delta})^{1-\beta}$$

the solution is

$$
\begin{aligned}
w &= w^* + \beta(p - w^*) \\
&= (1-\beta)rU + \beta p
\end{aligned}
$$

This is the wage that will be negotiated with a given U.

Now notice that rU is endogenous, and actually depends on w. In equilibrium we have to use (*) and this equation to get w. My calculation is:

$$w = \frac{(1-\beta)(r+\delta)b}{r+\delta+\beta\theta q(\theta)} + \frac{\beta(r+\delta) + \beta\theta q(\theta)}{r+\delta+\beta\theta q(\theta)}p$$

Substituting this into the inverse labor demand gets a second, upward-sloping relation between u and v. which is the "job creation" curve.

Beveridge Curve - Cobb-Douglas Match Function