Aggregate Seminar
Economics 137
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The Forward Discount Premium

Covered Interest Rate Parity says,

\[ \ln(1+i) = \ln(1+i^*) + \ln\left( \frac{F_{t+1}}{S} \right) \]

the forward discount equals the interest rate differential\(^1\). If covered interest rate parity
doesn’t hold, then arbitrage profits exist. Accept the covered interest parity as a fact.

Expected Interest Rate Parity\(^2\) is a theory that implies that \(E_t s_{t+1} = f_{t+1} \). A test of the
theory is the regression,

\[ \Delta_x_{t+1} = a + b(i - i^*) + u_{t+1}, \text{ or} \]
\[ \Delta_x_{t+1} = a + b(f_{t+1} - s_t) + e_{t+1} \]  \hspace{1cm} (1.1)

Under the null: \(a = 0, b = 1\), and the error \(e\) or \(u\) is unpredictable.

Profit

The empirical results generally show that expected interest rate parity is not a good
approximation to the data. On average the exchange rate does not depreciate enough to
compensate for the interest differential. Predictable expected excess returns exist.

How could one make money with this knowledge? A really simple rule is: Invest in the
country with the higher rate, ie,

if \((i-i^*) \geq 0\),
then, borrow abroad and invest at home, and
if \((i-i^*) < 0\),
then, borrow at home and invest abroad.

The realized profit from this rule is,

\(^1\) I use the notation from the project assignment description.
\(^2\) This assumes that the exchange rate is distributed log-normally.
\[ p^+ = (1 + i) - (1 + i^*) \frac{S_i}{S_i^*}; \quad i - i^* > 0 \]  
\[ p^- = -(1 + i) - (1 + i^*) \frac{S_i}{S_i^*}; \quad i - i^* < 0. \]

If the interest differential is greater than the realized exchange rate depreciation then, the profit is positive.

**Empirical Evidence**

**Data**

All the data come from Datastream. The data are monthly (measured on the 26th day of the month) for the exchange rate and the one-month forward rate (as collected by BBI). The data go from 9/26/93 to 9/26/03.

I used the forward discount \((f-s)\) as a proxy for the interest differential, \((i-i^*)\). And I used the log approximation to the profit calculation in equation (1.2), eg,

\[ p^+ @ (i - i^*) = \Delta x_{i-w} \]

**Australia**

The regression results do not support expected interest rate parity,

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.000202</td>
<td>0.002672</td>
<td>0.075411</td>
<td>0.9400</td>
</tr>
<tr>
<td>F_S</td>
<td>-0.288133</td>
<td>0.422369</td>
<td>-0.682182</td>
<td>0.4965</td>
</tr>
</tbody>
</table>

R-squared 0.003928  Mean dependent var -0.000247
Adjusted R-squared -0.004513  S.D. dependent var 0.028310
S.E. of regression 0.028374  Akaike info criterion -4.270195
Sum squared resid 0.094997  Schwarz criterion -4.223737
Log likelihood 258.2117  F-statistic 0.465372
Durbin-Watson stat 2.007813  Prob(F-statistic) 0.496461

The \(b\) coefficient is significantly less than one (p value of 1.5%).

Visual econometrics in a graph of the data confirm a weak relationship,
between the log change in the exchange rate and forward discount.

**Profit**

Can one make a profit betting against the theory?

Yes, on average.
Is it risky? Yes, the Sharpe ratio,

\[
Z^1 \frac{\text{mean}}{\text{std}} = 12\%
\]

is 12%. The Sharpe ratio for the S&P is about 6%. (Is the Sharpe ratio the correct measure of risk?)
Japan

A look at the raw data

shows the level of the exchange rate and the forward rate move closely together.
Test the theory

Dependent Variable: DLNS
Method: Least Squares
Date: 09/29/03   Time: 21:46
Sample(adjusted): 1993:09 2003:08
Included observations: 120 after adjusting endpoints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.003516</td>
<td>0.003826</td>
<td>-0.919084</td>
<td>0.3599</td>
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<tr>
<td>F_S</td>
<td>-1.008828</td>
<td>0.634076</td>
<td>-1.591020</td>
<td>0.1143</td>
</tr>
</tbody>
</table>

R-squared 0.021002     Mean dependent var 0.000450
Adjusted R-squared 0.012705   S.D. dependent var 0.031993
S.E. of regression 0.031789   Akaike info criterion -4.042865
Sum squared resid 0.119244   Schwarz criterion -3.996407
Log likelihood 244.5719   F-statistic 2.531343
Durbin-Watson stat 1.504952   Prob(F-statistic) 0.114281

The data fail to confirm the theory. The $b$ coefficient is far from one (p value < 1%)

Log changes in the exchange rate are very noisy relative to the forward discount.

Expected interest rate parity predicts a noisy relationship, since the forward rate is the expected future spot rate, $s_{t+1} = E s_{t+1} + e_{t+1} = f + e_{t+1}$. But the data reveal noise and no systematic relationship.
Profit: Can one make money betting against the theory?

Looks like it!

Sure can! Is it risky? The Sharpe ratio is 0.12.

LTCM made this bet and lost in 1998:8 and 1998:9. Was it unlucky? Here are the numbers
In August 1998 the monthly interest rate in the US was 0.1% higher than in Japan. So invest in the US. Bad move, the dollar depreciated by 7% (yen appreciated 7%) and LTCM lost 6.4% (a 2 std event) on the bet. And September was even worse. The interest differential was 0.4% in favor of the US, but the dollar depreciated by 13% (yen appreciated 13%), and LTCM lost 12%, (a 3.5 std outlier, and the minimum profit in the sample).

<table>
<thead>
<tr>
<th>Date</th>
<th>f-s</th>
<th>#yen/$</th>
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</thead>
<tbody>
<tr>
<td>1998:8</td>
<td>-0.001005</td>
<td>144.3200</td>
</tr>
<tr>
<td>1998:9</td>
<td>-0.004189</td>
<td>135.1500</td>
</tr>
<tr>
<td>1998:10</td>
<td>-0.004547</td>
<td>119.0400</td>
</tr>
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3 My exchange forward rate data and in #yen/$. So I treat Japan as the home country.
Data Warnings

<table>
<thead>
<tr>
<th>Series ID:</th>
<th>EXJPUS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source:</td>
<td>Board of Governors of the Federal Reserve System</td>
</tr>
<tr>
<td>Release:</td>
<td>G.5 Foreign Exchange Rates</td>
</tr>
<tr>
<td>Seasonal Adjustment:</td>
<td>Not Applicable</td>
</tr>
<tr>
<td>Frequency:</td>
<td>Monthly</td>
</tr>
<tr>
<td>Units:</td>
<td>Japanese Yen to One U.S. Dollar</td>
</tr>
<tr>
<td>Date Range:</td>
<td>1971-01-01 to 2003-08-01</td>
</tr>
<tr>
<td>Last Updated:</td>
<td>2003-09-02</td>
</tr>
<tr>
<td>Notes:</td>
<td>Averages of daily figures. Noon buying rates in New York City for cable transfers payable in foreign currencies.</td>
</tr>
</tbody>
</table>

Latest Observations:

This is a very nice description and picture. But notice that the monthly data are the average of the daily data. Actual trades take place on a day and profits are realized one month later. Daily movements during the month don’t matter. Averaged data is not appropriate for testing most models.
Profit

Series: P0
Sample 1993:09 2003:08
Observations 120

Mean -0.004382
Median -0.007961
Maximum 0.122736
Minimum -0.098420
Std. Dev. 0.032974
Skewness 0.608072
Kurtosis 4.806089
Jarque-Bera 23.70483
Probability 0.000007