Economics 101A
(Lecture 5, Revised)

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Outline

1. Properties of Preferences (continued)

2. From Preferences to Utility (and vice versa)

3. Common Utility Functions

4. (Utility maximization)
1 Properties of Preferences (ctd)

- Indifference relation $\sim$: $x \sim y$ if $x \succeq y$ and $y \succeq x$

- Strict preference: $x \succ y$ if $x \succeq y$ and not $y \succeq x$

- Exercise. If $\succeq$ is rational,
  - $\succ$ is transitive
  - $\sim$ is transitive
  - Reflexive property of $\succeq$. For all $x$, $x \succeq x$. 
• Other features of preferences

• Preference relation $\succeq$ is:

  – *monotonic* if $x \geq y$ implies $x \succeq y$.

  – *strictly monotonic* if $x \geq y$ and $x_j > y_j$ for some $j$ implies $x \succ y$.

  – *convex* if for all $x$, $y$, and $z$ in $X$ such that $x \succeq z$ and $y \succeq z$, then $tx + (1 - t)y \succeq z$ for all $t$ in $[0, 1]$
2 From preferences to utility

- Nicholson, Ch. 3

- Economists like to use utility functions $u : X \rightarrow R$

- $u(x)$ is ‘liking’ of good $x$

- $u(a) > u(b)$ means: I prefer $a$ to $b$.

- **Def.** Utility function $u$ represents preferences $\succeq$ if, for all $x$ and $y$ in $X$, $x \succeq y$ if and only if $u(x) \geq u(y)$.

- **Theorem.** If preference relation $\succeq$ is rational and continuous, there exists a continuous utility function $u : X \rightarrow R$ that represents it.
Proof for case $X = R^2_+$ and $\succeq$ strongly monotonic.

- Define $u(x) =$?

- Consider the points in the diagonal, $(t, t)$

- Set $\{t : (t, t) \succeq x\}$ is non-empty by monotonicity

- Set $\{t : x \succeq (t, t)\}$ is non-empty by monotonicity

- Both sets are closed by continuity

- (Connected set $X$: $A \subset X$ closed, $B \subset X$ closed, and $A \cup B = X \implies A \cap B$ non-empty)

- By connectedness of $R$, the two sets have non-empty intersection $\implies \exists t_x$ such that $(t_x, t_x) \sim x$. Define $u(x) = t_x$. 


- Does \( u \) represent \( \succeq \)?

- \( x \succeq y \) implies \((u(x), u(x)) \sim x \succeq y \sim (u(y), u(y)) \Rightarrow \)
[by transitivity] \((u(x), u(x)) \succeq (u(y), u(y)) \Rightarrow \)
[by monotonicity] \( u(x) \geq u(y) \)

- Similarly can prove other direction (exercise!)

- (We do not prove continuity of \( u(x) \))
• Utility function representing $\succeq$ is not unique

• Take $\exp(u(x))$

• $u(a) > u(b) \iff \exp(u(a)) > \exp(u(b))$

• If $u(x)$ represents preferences $\succeq$ and $f$ is a strictly increasing function, then $f(u(x))$ represents $\succeq$ as well.

• If preferences are represented from a utility function, are they rational?
  
  – completeness

  – transitivity
• Indifference curves: \( u(x_1, x_2) = \tilde{u} \)

• They are just implicit functions! \( u(x_1, x_2) - \tilde{u} = 0 \)

\[
\frac{dx_2}{dx_1} = -\frac{U'_x}{U'_{x_2}} = MRS
\]

• Indifference curves for:

  – monotonic preferences;

  – strictly monotonic preferences;

  – convex preferences
3 Common utility functions

- Nicholson, Ch. 3, pp. 80–84

1. Cobb-Douglas preferences: \( u(x_1, x_2) = x_1^\alpha x_2^{1-\alpha} \)
   
   - \( MRS = -\alpha x_1^{a-1} x_2^{1-\alpha} / (1-a) x_1^\alpha x_2^{-\alpha} = \frac{\alpha x_2}{1-\alpha x_1} \)

2. Perfect substitutes: \( u(x_1, x_2) = \alpha x_1 + \beta x_2 \)
   
   - \( MRS = -\alpha / \beta \)
3. Perfect complements: \( u(x_1, x_2) = \min(\alpha x_1, \beta x_2) \)

- \( MRS \) discontinuous at \( x_2 = \frac{\alpha}{\beta} x_1 \)

4. Constant Elasticity of Substitution: \( u(x_1, x_2) = (\alpha x_1^\rho + \beta x_2^\rho)^{1/\rho} \)

- \( MRS = -\frac{\alpha}{\beta} \left(\frac{x_1}{x_2}\right)^{\rho-1} \)
- if \( \rho = 1 \), then...
- if \( \rho = 0 \), then...
- if \( \rho \to +\infty \), then...