Outline

1. Investment in Risky Asset

2. Measures of Risk Aversion

3. Time Consistency

4. Time Inconsistency
1 Investment in Risk Asset

- Individual has:
  - wealth $w$
  - utility function $u$, with $u' > 0$

- Two possible investments:
  - Asset B (bond) yields return 1 for each dollar
  - Asset S (stock) yields uncertain return $(1 + r)$:
    * $r = r_+ > 0$ with probability $p$
    * $r = r_- < 0$ with probability $1 - p$
    * $Er = pr_+ + (1 - p)r_- > 0$

- Share of wealth invested in stock $S = \alpha$
• Individual maximization:

\[
\max_{\alpha} (1 - p) u(w [(1 - \alpha) + \alpha (1 + r_-)]) + \\
+pu(w [(1 - \alpha) + \alpha (1 + r_+)]) \\
s.t. 0 \leq \alpha \leq 1
\]

• Case of risk aversion: \( u'' < 0 \)

• Assume \( 0 \leq \alpha^* \leq 1 \), check later

• First order conditions:

\[
0 = (1 - p) (wr_-) u' (w [1 + \alpha r_-]) + \\
+p (wr_+ ) u' (w [1 + \alpha r_+])
\]

• Can \( \alpha^* = 0 \) be solution?
• Solution is $\alpha^* > 0$ (positive investment in stock)

• Exercise: Check s.o.c.
2 Measures of Risk Aversion

• Nicholson, Ch. 8, pp. 207–210.

• How risk averse is an individual?

• Two measures:

  – Absolute Risk Aversion $r_A$:

    \[ r_A = -\frac{u''(x)}{u'(x)} \]

  – Relative Risk Aversion $r_R$:

    \[ r_R = -\frac{u''(x)}{u'(x)}x \]

• Examples in the Problem Set
3 Time consistency

• Nicholson, Ch. 23, pp. 629–633. (Certainty case only)

• Intertemporal choice

• Three periods, $t = 0$, $t = 1$, and $t = 2$

• At each period $i$, agents:
  
  – have income $M_i^t = M_i + \text{savings/debts from previous period}$

  – choose consumption $c_i$;

  – can save/borrow $M_i^t - c_i$

  – no borrowing in last period: at $t = 2$ $M_2^t = c_2$
• Utility function at $t = 0$

$$u(c_0, c_1, c_2) = U(c_0) + \frac{1}{1 + \delta} EU(c_1) + \frac{1}{(1 + \delta)^2} EU(c_2)$$

• Utility function at $t = 1$

$$u(c_1, c_2) = U(c_1) + \frac{1}{1 + \delta} EU(c_2)$$

• Utility function at $t = 2$

$$u(c_2) = U(c_2)$$

• $U' > 0$, $U'' < 0$
• Question: Do preferences of agent in period 0 agree with preferences of agent in period 1?

• Period 1.

• Budget constraint at $t = 1$:

$$c_1 + \frac{1}{1 + r}c_2 \leq M_1' + \frac{1}{1 + r}M_2$$

• Maximization problem:

$$\max U(c_1) + \frac{1}{1 + \delta}EU(c_2)$$

$s.t. c_1 + \frac{1}{1 + r}c_2 \leq M_1' + \frac{1}{1 + r}M_2$

• First order conditions:

• Ratio of f.o.c.s:

$$\frac{U'(c_1)}{EU'(c_2)} = \frac{1 + r}{1 + \delta}$$
• Back to **period 0**.

• Agent at time 0 can commit to consumption at time 1 as function of stochastic income $M_1$.

• Anticipated budget constraint at $t = 1$:

$$c_1 + \frac{1}{1 + r}c_2 \leq M'_1 + \frac{1}{1 + r}M_2$$

• Maximization problem:

$$\max U(c_0) + \frac{1}{1 + \delta}U(c_1) + \frac{1}{(1 + \delta)^2}EU(c_2)$$

$$s.t. \ c_1 + \frac{1}{1 + r}c_2 \leq M'_1 + \frac{1}{1 + r}M_2$$

• First order conditions:

• Ratio of f.o.c.s:

$$\frac{U''(c_1)}{EU''(c_2)} = \frac{1 + r}{1 + \delta}$$
• The two conditions coincide!

• **Time consistency.** Plans for future coincide with future actions.

• To see why, rewrite utility function \( u(c_0, c_1, c_2) \):

\[
U(c_0) + \frac{1}{1+\delta}U(c_1) + \frac{1}{(1+\delta)^2}EU(c_2) \\
= \ U(c_0) + \frac{1}{1+\delta} \left[ U(c_1) + \frac{1}{1+\delta}EU(c_2) \right]
\]

• Expression in brackets coincides with utility at \( t = 1 \)

• Is time consistency right?
  
  – addictive products (alcohol, drugs);

  – good actions (exercising, helping friends);

  – immediate gratification (shopping, credit card borrowing)
4 Time Inconsistency

- Alternative specification (Akerlof, 1991; Laibson, 1997; O’Donoghue and Rabin, 1999)

- Utility at time $t$ is $u(c_t, c_{t+1}, c_{t+2})$:
  \[ u(c_t) + \frac{\beta}{1 + \delta} u(c_{t+1}) + \frac{\beta}{(1 + \delta)^2} u(c_{t+2}) + \ldots \]

- Discount factor is
  \[ 1, \frac{\beta}{1 + \delta}, \frac{\beta}{(1 + \delta)^2}, \frac{\beta}{(1 + \delta)^3}, \ldots \]
  instead of
  \[ 1, \frac{1}{1 + \delta}, \frac{1}{1 + \delta}, \frac{1}{(1 + \delta)^2}, \frac{1}{(1 + \delta)^3}, \ldots \]

- What is the difference?

- Immediate gratification: $\beta < 1$
• Back to our problem: **Period 1**.

• Maximization problem:

\[
\max U(c_1) + \frac{\beta}{1 + \delta}EU(c_2)
\]

\[
s.t. \ c_1 + \frac{1}{1 + r}c_2 \leq M'_1 + \frac{1}{1 + r}M_2
\]

• First order conditions:

• Ratio of f.o.c.s:

\[
\frac{U'(c^*_1)}{EU'(c^*_2)} = \beta \frac{1 + r}{1 + \delta}
\]
• Now, **period 0** with commitment.

• Maximization problem:

\[
\max U(c_0) + \frac{\beta}{1 + \delta} U(c_1) + \frac{\beta}{(1 + \delta)^2} EU(c_2)
\]

s.t. \[c_1 + \frac{1}{1 + r} c_2 \leq M_1' + \frac{1}{1 + r} M_2\]

• First order conditions:

• Ratio of f.o.c.s:

\[
\frac{U'(c_1^*,c)}{EU'(c_2^*,c)} = \frac{1 + r}{1 + \delta}
\]

• The two conditions differ!

• Time inconsistency: \[c_1^*,c < c_1^* \text{ and } c_2^*,c > c_2^*\]

• The agent allows him/herself too much immediate consumption and saves too little
• Ok, we agree. but should we study this as economists?

• YES!
  – One trillion dollars in credit card debt;
  – Most debt is in teaser rates;
  – Two thirds of Americans are overweight or obese;
  – $10bn health-club industry

• Is this testable?
  – In the laboratory?
  – In the field?
5 Next lecture and beyond

- Th:
  - Finish Time Inconsistency
  - Begin Production
  - Returns to scale
  - Cost minimization