Outline

1. Health Club Attendance

2. Production: Introduction

3. Production Function

4. Returns to Scale

5. Two-step Cost Minimization
1 Health Club Attendance

• Health club industry study (DellaVigna and Malmendier, 2002)

• 3 health clubs

• Data on attendance from swiping cards

• Choice of contracts:
  – Monthly contract with average price of $75
  – 10-visit pass for $100

• Consider users that choose monthly contract. Attendance?
• Attend on average 4.8 times per month

• Pay on average over $17

• Average delay of 2.2 months ($185) between last attendance and contract termination

• Over membership, user could have saved $700 by paying per visit
• Health club attendance:
  
  – immediate cost $c$
  
  – delayed benefit $b$

• At sign-up (attend tomorrow):

$$NB^t = -\frac{\beta}{1+\delta}c + \frac{\beta}{(1+\delta)^2}b$$

• Plan to attend if $NB^t > 0$

$$c < \frac{1}{(1+\delta)}b$$
• Once moment to attend comes:

\[ NB = -c + \frac{\beta}{(1 + \delta)}b \]

• Attend if \( NB > 0 \)

\[ c < \frac{\beta}{(1 + \delta)}b \]
• Interpretations?

• Users are buying a commitment device

• User underestimate their future self-control problems:
  – They overestimate future attendance
  – They delay cancellation
2 Production: Introduction

• Second half of the economy. Production

• Example. Ford and the Minivan (Petrin, 2002):
  – Ford had idea: "Mini/Max" (early '70s)
  – Did Ford produce it?
    – No!
  – Ford was worried of cannibalizing station wagon sector
  – Chrysler introduces Dodge Caravan (1984)
  – Chrysler: $1.5bn profits (by 1987)!
• Why need separate treatment?

• Perhaps firms maximize utility...

• ...we can be more precise:
  – Competition
  – Institutional structure
3 Production Function

- Nicholson, Ch. 11

- Production function: \( y = f (z) \). Function \( f : \mathbb{R}_+^n \rightarrow \mathbb{R}_+ \)

- Inputs \( z = (z_1, z_2, ..., z_n) \): labor, capital, land, human capital

- Output \( y \): Minivan, Intel Pentium III, mangoes (Philippines)

- Properties of \( f \):
  - no free lunches: \( f (0) = 0 \)
  - positive marginal productivity: \( f_i^1 (z) > 0 \)
  - decreasing marginal productivity: \( f_{i,i}^{11} (z) < 0 \)
• Isoquants $Q(y) = \{ x | f(x) = y \}$

• Set of inputs $z$ required to produce quantity $y$

• Special case. Two inputs:
  - $z_1 = L$ (labor)
  - $z_2 = K$ (capital)

• Isoquant: $f(L, K) - y = 0$

• Slope of isoquant $dK/dL = MRTS$
• Convex production function if convex isoquants

• Reasonable: combine two technologies and do better!

• Mathematically, $\frac{d^2K}{d^2L} =$
4 Returns to Scale

- Effect of increase in labor: $f'_L$

- Increase of all inputs: $f(tz)$ with $t$ scalar, $t > 1$
  [REVISED, notice $t > 1$]

- How much does input increase?
  - Decreasing returns to scale: for all $z$ and $t > 1$
    [REVISED, notice $t > 1$]
    $$f(tz) < tf(z)$$

  - Constant returns to scale: for all $z$ and $t > 1$
    [REVISED, notice $t > 1$]
    $$f(tz) = tf(z)$$
Increasing returns to scale: for all $z$ and $t > 1$, [REVISED, notice $t > 1$]

$$f(tz) > tf(z)$$
• Example: \( y = f(K, L) = AK^\alpha L^\beta \)

• Marginal product of labor: \( f'_L = \)

• Decreasing marginal product of labor: \( f''_L = \)

• \( MRTS = \)

• Convex isoquant?

• Returns to scale: \( f(tK, tL) = A(tK)^\alpha (tL)^\beta = t^{\alpha+\beta} AK^\alpha L^\beta = t^{\alpha+\beta} f(K, L) \)
5 Two-step Cost minimization

- Nicholson, Ch. 12

- Objective of firm: Produce output that generates maximal profit.

- Decompose problem in two:
  - Given production level $y$, choose cost-minimizing combinations of inputs
  - Choose optimal level of $y$.

- *First step.* Cost-Minimizing choice of inputs
• Two-input case: Labor, Capital

• Input prices:
  
  – Wage $w$ is price of $L$
  
  – Interest rate $r$ is rental price of capital $K$

• Expenditure on inputs: $wL + rK$

• Firm objective function:

$$\min wL + rK$$

$$s.t. f (L, K) \geq y$$
• Compare with expenditure minimization for consumers

• First order conditions:

\[ w - \lambda f_L' = 0 \]

and

\[ r - \lambda f_K' = 0 \]

• Rewrite as

\[ \frac{f_L'(L^*, K^*)}{f_K'(L^*, K^*)} = \frac{w}{r} \]

• MRTS (slope of isoquant) equals ratio of input prices
• Graphical interpretation
• Derived demand for inputs:

\[- L = L^* (w, r, y) \]

\[- K = K^* (w, r, y) \]

• Value function at optimum is **cost function**:

\[ c (w, r, y) = wL^* (r, w, y) + rK^* (r, w, y) \]
• **Second step.** Given cost function, choose optimal quantity of \( y \) as well

• Price of output is \( p \).

• Firm’s objective:

\[
\max py - c(w, r, y)
\]

• First order condition:

\[
p - c'_y(w, r, y) = 0
\]

• Price equals marginal cost – very important!
6 Next Lecture

• Continue Cost Minimization

• Solve an Example

• Cases in which s.o.c. are not satisfied

• Start Profit Maximization