Outline

1. Geometry of Cost Curves

2. Supply Function

3. Short-run Cost Minimization

4. One-step Profit Maximization

5. Introduction to Market Equilibrium
1 Geometry of cost curves


- Marginal costs $MC = \partial c/\partial y \rightarrow$ Cost minimization
  
  $$p = MC = \partial c(w, r, y)/\partial y$$

- Average costs $AC = c/y \rightarrow$ Does firm break even?
  
  $$\pi = py - c(w, r, y) > 0 \text{ iff }$$
  
  $$\pi/y = p - c(w, r, y)/y > 0 \text{ iff }$$
  
  $$c(w, r, y)/y = AC < p$$

- **Supply function.** Portion of marginal cost $MC$ above average costs. (price equals marginal cost)
• Assume only 1 input (expenditure minimization is trivial)

• **Case 1.** Production function. \( y = L^\alpha \)

  – Cost function? (cost of input is \( w \)):
    \[
    c(w, y) = wL^*(w, y) = wy^{1/\alpha}
    \]

  – Marginal cost?
    \[
    \frac{\partial c(w, y)}{\partial y} = \frac{1}{\alpha}wy^{(1-\alpha)/\alpha}
    \]

  – Average cost \( c(w, y)/y \)?
    \[
    \frac{c(w, y)}{y} = \frac{wy^{1/\alpha}}{y} = wy^{(1-\alpha)/\alpha}
    \]
• **Case 1a.** $\alpha > 1$. Plot production function, total cost, average and marginal. Supply function?

• **Case 1b.** $\alpha = 1$. Plot production function, total cost, average and marginal. Supply function?

• **Case 1c.** $\alpha < 1$. Plot production function, total cost, average and marginal. Supply function?
• **Case 2.** *Non-convex technology.* Plot production function, total cost, average and marginal. Supply function?

• **Case 3.** *Technology with setup cost.* Plot production function, total cost, average and marginal. Supply function?
2 Supply Function

• Supply function: \( y^* = y^* (w, r, p) \)

• What happens to \( y^* \) as \( p \) increases?

• Is the supply function upward sloping?

• Remember f.o.c:

\[
p - c'_y (w, r, y) = 0
\]

• Implicit function:

\[
\frac{\partial y^*}{\partial p} = -\frac{1}{-c''_{y,y} (w, r, y)} > 0
\]

as long as s.o.c. is satisfied.
• Yes! Supply function is upward sloping.
3 Short-run Cost Minimization

• So far, we assumed flexibility in choose of all inputs.

• Is this realistic?
  – In long-run, yes. Can adjust machines, land,…
  – But… in the long-run, we are all dead! (Keynes)
  – In short-run, no. Capital and land are fixed.

• Short-run cost minimization: $K$ fixed at $\overline{K}$.

• Firm’s objective function:

  \[
  \min_{L} wL + r\overline{K} \\
  \text{s.t. } f(L, \overline{K}) \geq y
  \]

• Capital $\overline{K}$ is a constant.
• Solution:

\[ L^* = L_{SR}^*(r, w, y|K) \]

• Short-run cost function

\[ c_{SR}(r, w, y|\overline{K}) = wL_{SR}^*(r, w, y|\overline{K}) + r\overline{K} \]

• Exercise: Show \( c_{SR}(r, w, y|\overline{K}) > c(r, w, y) \)

• Graphically,
4 One-step Profit Maximization

• Nicholson, Ch. 13, pp. 346–350.

• One-step procedure: maximize profits

• Perfect competition. Price $p$ is given
  – Firms are small relative to market
  – Firms do not affect market price $p_M$

  – Will firm produce at $p > p_M$?
  – Will firm produce at $p < p_M$?
  – $\implies p = p_M$
- Revenue: \( py = pf(L, K) \)

- Cost: \( wL + rK \)

- Profit \( pf(L, K) - wL - rK \)
• Agent optimization:

\[
\max_{L,K} pf(L, K) - wL - rK
\]

• First order conditions:

\[
 pf'_L(L, K) - w = 0
\]

and

\[
 pf'_K(L, K) - r = 0
\]

• Second order conditions? \( pf''_{L,L}(L, K) < 0 \) and

\[
|H| = \begin{vmatrix}
 pf''_{L,L}(L, K) & pf''_{L,K}(L, K) \\
 pf''_{L,K}(L, K) & pf''_{K,K}(L, K)
\end{vmatrix} =
\]

\[
= p^2 \left[ f''_{L,L} f''_{K,K} - \left( f''_{L,K} \right)^2 \right] > 0
\]

• Need \( f''_{L,K} \) not too large for maximum
- Comparative statics with respect to to $p, w,$ and $r$.

- What happens if $w$ increases?

\[
\frac{\partial L^*}{\partial w} = -\begin{vmatrix}
-1 & pf''_{L,K}(L,K) \\
0 & pf''_{K,K}(L,K)
\end{vmatrix} < 0
\]

and

\[
\frac{\partial L^*}{\partial r} =
\]

- Sign of $\frac{\partial L^*}{\partial r}$ depends on $f''_{L,K}$. 