Outline

1. One-Step Profit Maximization

2. Introduction to Market Equilibrium

3. Aggregation

4. Maket Equilibrium in Short-Run

5. Comparative Statics of Equilibrium
• Pages covered on Nicholson so far include (not necessarily exhaustive):

  – Nicholson, Ch. 8, pp. 98–110. (Expected Utility)

  – Nicholson, Ch. 11, pp. 268–278, 280–285


1 One-Step Profit Maximization

• Firm optimization:

$$\max_{L, K} pf(L, K) - wL - rK$$

• First order conditions:

$$pf_L'(L, K) - w = 0$$

and

$$pf_K'(L, K) - r = 0$$

• Second order conditions? $$pf''_{L,L}(L, K) < 0$$, and

$$|H| = \begin{vmatrix} pf''_{L,L}(L, K) & pf''_{L,K}(L, K) \\ pf''_{L,K}(L, K) & pf''_{K,K}(L, K) \end{vmatrix} =$$

$$= p^2 \left[ pf''_{L,L} pf''_{K,K} - (pf''_{L,K})^2 \right] > 0$$
• Comparative statics with respect to to $p$, $w$, and $r$.

• What happens if $w$ increases?

$$\frac{\partial L^*}{\partial w} = -\frac{\begin{vmatrix}
-1 & pf''_{L,K}(L,K) \\
0 & pf''_{K,K}(L,K)
\end{vmatrix}}{pf''_{L,L}(L,K) pf''_{K,K}(L,K) - pf''_{L,K}(L,K) pf''_{L,K}(L,K)} < 0$$

and

$$\frac{\partial L^*}{\partial p} =$$

• $\partial L^*/\partial p > 0$ if $f''_{L,K} > 0$. 
2 Introduction to Market Equilibrium

- Nicholson, Ch. 14, pp. 368–382.

- Two ways to analyze firm behavior:
  - Two-Step Cost Minimization
  - One-Step Profit Maximization

- What did we learn?
  - Optimal demand for inputs $L^*, K^*$ (see above)
  - Optimal quantity produced $y^*$
• **Supply function.** \( y = y^* (p, w, r) \)

  – From profit maximization:
    \[
    y = f \left( L^* (p, w, r), K^* (p, w, r) \right)
    \]

  – From cost minimization:
    \[
    MC \text{ curve above } AC \] [REVISED]

  – Supply function is increasing in \( p \)

• **Market Equilibrium.** Equate demand and supply.

• **Aggregation?**

• **Industry supply function!**
3 Aggregation

3.1 Producers aggregation

- $J$ companies, $j = 1, ..., J$, producing good $i$

- Company $j$ has supply function

\[ y_i^j = y_i^{j*}(p_i, w, r) \]

- Industry supply function:

\[ Y_i(p_i, w, r) = \sum_{j=1}^{J} y_i^{j*}(p_i, w, r) \]

- Graphically,
3.2 Consumer aggregation

- Nicholson, Ch. 7, pp. 172–176

- One-consumer economy

- Utility function $u(x_1, \ldots, x_n)$

- Prices $p_1, \ldots, p_n$

- Maximization $\implies$

\[
\begin{align*}
x_1^* & = x_1^*(p_1, \ldots, p_n, M), \\
& \vdots \\
x_n^* & = x_n^*(p_1, \ldots, p_n, M).
\end{align*}
\]
• Focus on good $i$. Fix prices $p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n$ and $M$

• **Single-consumer demand function:**

$$x_i^* = x_i^* (p_i|p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n, M)$$

• What is sign of $\partial x_i^*/\partial p_i$?

• Negative if good $i$ is normal

• Negative or positive if good $i$ is inferior
• **Aggregation:** $J$ consumers, $j = 1, \ldots, J$

• Demand for good $i$ by consumer $j$:

$$x_{i}^{j*} = x_{i}^{j*} \left( p_{1}, \ldots, p_{n}, M^{j} \right)$$

• Market demand $X_{i}$:

$$X_{i} \left( p_{1}, \ldots, p_{n}, M^{1}, \ldots, M^{J} \right) = \sum_{j=1}^{J} x_{i}^{j*} \left( p_{1}, \ldots, p_{n}, M^{j} \right)$$

• Graphically,
• Notice: market demand function depends on distribution of income $M^J$

• Market demand function $X_i$:
  
  – Consumption of good $i$ as function of prices $p$
  
  – Consumption of good $i$ as function of income distribution $M^j$
4 Market Equilibrium in the Short-Run

- Nicholson, Ch. 14, pp. 368–382.

- What is equilibrium price $p_i$?

- Magic of the Market...

- Equilibrium: No excess supply, No excess demand

- Prices $p^*$ equates demand and supply of good $i$:

$$Y^* = Y^S_i (p^*_i, w, r) = X_i^D (p^*_1, ..., p^*_n, M^1, ..., M^J)$$
• Graphically,

• Notice: in short-run firms can make positive profits
• Comparative statics exercises with endogenous price $p_i$:
  
  – increase in wage $w$ or interest rate $r$:

  – change in income distribution
5 Comparative statics of equilibrium

• Supply and Demand function of parameter $\alpha$
  
  - $Y_i^S (p_i, w, r, \alpha)$
  
  - $X_i^D (p, M, \alpha)$

• How does $\alpha$ affect $p^*$ and $Y^*$?

• Comparative statics with respect to $\alpha$

• Equilibrium:
  
  $$Y_i^S (p_i, w, r, \alpha) = X_i^D (p, M, \alpha)$$
• Can write equilibrium as implicit function:

\[ Y^S_i (p_i, w, r, \alpha) - X^D_i (p, M, \alpha) = 0 \]

• What is \( dp^*/d\alpha \)?

• Implicit function theorem:

\[
\frac{\partial p^*}{\partial \alpha} = -\frac{\partial Y^S}{\partial p} \frac{\partial X^D}{\partial p} - \frac{\partial Y^S}{\partial \alpha} \frac{\partial X^D}{\partial \alpha}
\]

• What is sign of denominator?

• Sign of \( \partial p^*/\partial \alpha \) is negative of sign of numerator
• How do we interpret magnitudes of $\partial p^*/\partial \alpha$?

• Result depends on unit of measures

• Can we write $\partial p^*/\partial \alpha$ in a unit-free way?

• Yes! Use elasticities.

• Elasticity of $x$ with respect to parameter $p$ is

$$\varepsilon_{x,p} = \frac{\partial x}{\partial p} \frac{p}{x}$$

• Interpretation: Percent response in $x$ to percent change in $p$:

$$\varepsilon_{x,p} = \frac{\partial x}{\partial p} \frac{p}{x} = \frac{dx}{x} \frac{dp}{p}$$

• Exercise:

$$\varepsilon_{x,p} = \frac{\partial \ln x}{\partial \ln p}$$
• Use elasticities to rewrite response of $p$ to change in $\alpha$:

$$\frac{\partial p^* \alpha}{\partial \alpha \ p} = -\frac{\left( \frac{\partial Y^S}{\partial \alpha} - \frac{\partial X^D}{\partial \alpha} \right) \frac{\alpha}{Y}}{\left( \frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p} \right) \frac{p}{Y}}$$

or

$$\varepsilon_{p,\alpha} = -\frac{\varepsilon_{S,\alpha} - \varepsilon_{D,\alpha}}{\varepsilon_{S,p} - \varepsilon_{D,p}}$$

• We are likely to know elasticities from empirical studies.