Outline

1. Elasticities

2. Comparative Statics of SR Equilibrium II

3. Response to Taxes
1 Elasticities

- Elasticity of $x$ with respect to parameter $p$ is
  \[ \varepsilon_{x,p} = \frac{\partial x}{\partial p} \]

- Interpretation: Percent response in $x$ to percent change in $p$:
  \[ \varepsilon_{x,p} = \frac{\partial x}{\partial p} \frac{1}{x} = \lim_{dp \to 0} \frac{x(p + dp) - x(p)}{dp} = \lim_{dp \to 0} \frac{dx}{dx} \frac{dp}{p} \]
  where $dx \equiv x(p + dp) - x(p)$.

- Now, show
  \[ \varepsilon_{x,p} = \frac{\partial \ln x}{\partial \ln p} \]

- Notice: This makes sense only for $x > 0$ and $p > 0$
Consider function

\[ x = f(p) \]

Rewrite as

\[ \ln(x) = \ln(f(p)) = \ln(f(e^{\ln(p)})) \]

Define \( \hat{x} = \ln(x) \) and \( \hat{p} = \ln(p) \)

This implies

\[ \hat{x} = \ln(f(e^\hat{p})) \]

Get

\[ \frac{\partial \hat{x}}{\partial \hat{p}} = \frac{\partial \ln x}{\partial \ln p} = \frac{1}{f(e^\hat{p})} \frac{\partial f(e^\hat{p})}{\partial \hat{p}} e^\hat{p} = \frac{\partial x p}{\partial p x} \]
• Example with Cobb-Douglas utility function

• \( U(x, y) = x^\alpha y^{1-\alpha} \) implies solutions

\[
x^* = \alpha \frac{M}{p_x}, \quad y^* = (1 - \alpha) \frac{M}{p_y}
\]

• Elasticity \( \varepsilon_{x,p_x} \):

\[
\varepsilon_{x,p_x} = \frac{\partial x^* p_x}{\partial p_x x^*} = -\alpha M \frac{p_x}{(p_x)^2} \frac{1}{\alpha} = -1
\]

• \( \varepsilon_{x,p_y} = 0 \)
2 Comparative statics of SR equilibrium II

• Supply and Demand function of parameter $\alpha$:

$$Y_i^S(p_i, w, r, \alpha)$$

$$X_i^D(p, M, \alpha)$$

• How does $\alpha$ affect $p^*$ and $Y^*$?

• Comparative statics with respect to $\alpha$

• Equilibrium:

$$Y_i^S(p_i, w, r, \alpha) = X_i^D(p, M, \alpha)$$
• Can write equilibrium as implicit function:
\[ Y_i^S(p_i, w, r, \alpha) - X_i^D(p, M, \alpha) = 0 \]

• Implicit function theorem:
\[
\frac{\partial p^*}{\partial \alpha} = -\frac{\frac{\partial Y^S}{\partial \alpha}}{\frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p}}
\]

• Use elasticities to rewrite response of \( p \) to change in \( \alpha \):
\[
\frac{\partial p^* \alpha}{\partial \alpha \ p} = -\left( \frac{\frac{\partial Y^S}{\partial \alpha} - \frac{\partial X^D}{\partial \alpha}}{\frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p}} \right) \frac{\alpha}{Y}
\]

or (using fact that \( X^D* = Y^S* \))
\[
\varepsilon_{p,\alpha} = -\frac{\varepsilon_{S,\alpha} - \varepsilon_{D,\alpha}}{\varepsilon_{S,p} - \varepsilon_{D,p}}
\]

• We are likely to know elasticities from empirical studies.
3 Response to taxes

- Nicholson, Ch. 15, pp. 407–408

- Per-unit tax $t$

- Write price $p_i$ as price including tax

- Supply: $Y_i^S (p_i - t, w, r)$

- Demand: $X_i^D (p, M)$
  \[ Y_i^S (p_i - t, w, r) - X_i^D (p, M) = 0 \]

- What is $dp^*/dt$?
• Comparative statics:

\[
\frac{\partial p^*}{\partial t} = -\frac{\partial Y^S}{\partial p} \frac{\partial Y^S}{\partial t} = -\frac{\partial Y^S}{\partial p} \frac{\partial X^D}{\partial p} = -\frac{\partial Y^S}{\partial p} \frac{p}{X} = \left( \frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p} \right) \frac{p}{X} = \frac{\varepsilon_{S,p}}{\varepsilon_{S,p} - \varepsilon_{D,p}}
\]

• How about price received by suppliers \( p^* - t \)?

\[
\frac{\partial (p^* - t)}{\partial t} = \frac{\partial Y^S}{\partial p} \frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p} - 1 = \frac{\varepsilon_{D,p}}{\varepsilon_{S,p} - \varepsilon_{D,p}}
\]
• *Inflexible Supply.* Supply curve vertical \((\varepsilon_{S,p} = 0)\)

- Producers bear burden of tax [REVISED]

• *Flexible Supply.* Supply curve horizontal \((\varepsilon_{S,p} \to \infty)\)

- Consumers bear burden of tax [REVISED]
• Inflexible demand. Demand curve vertical ($\varepsilon_{D,p} = 0$)?

• Consumers bear burden [REVISED]

• General lesson: Most elastic side bears larger part of burden

• What happens with a subsidy ($t < 0$)?
• What happens to quantity sold?

• Use demand curve:

\[
\frac{\partial X^D}{\partial t} = \frac{\partial X^D}{\partial p^*} \frac{\partial p^*}{\partial t}
\]

and use expression for \( \frac{\partial p^*}{\partial t} \) above.