Outline

1. Game Theory

2. Oligopoly: Cournot

3. Oligopoly: Bertrand

4. Dynamic Games
1 Game Theory

- Nicholson, Ch. 10, pp. 246–255.

- Definitions:
  
  - Players: $1, \ldots, I$
  
  - Strategy $s_i \in S_i$

  - Payoffs: $U_i(s_i, s_{-i})$
Example: Prisoner’s Dilemma

- $I = 2$

- $s_i = \{D, ND\}$

- Payoffs matrix:

\[
\begin{array}{c|ccc}
   & D & ND \\
\hline
D & -4, -4 & -1, -5 \\
ND & -5, -1 & -2, -2 \\
\end{array}
\]
• What prediction?

• Maximize sum of payoffs

• Choose dominant strategies

• **Equilibrium in dominant strategies**

• Strategies \( s^* = (s^*_i, s^*_{-i}) \) are an Equilibrium in dominant strategies if

\[
U_i(s^*_i, s_{-i}) \geq U_i(s_i, s_{-i})
\]

for all \( s_i \in S_i \), for all \( s_{-i} \in S_{-i} \) and all \( i = 1, \ldots, I \)
• Battle of the Sexes game:

<table>
<thead>
<tr>
<th></th>
<th>Ballet</th>
<th>Football</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ballet</td>
<td>2, 1</td>
<td>0, 0</td>
</tr>
<tr>
<td>Football</td>
<td>0, 0</td>
<td>1, 2</td>
</tr>
</tbody>
</table>

• No dominant strategies

• Nash Equilibrium.

• Strategies $s^* = (s^*_i, s^*_{-i})$ are a Nash Equilibrium if

$$U_i(s^*_i, s^*_{-i}) \geq U_i(s_i, s^*_{-i})$$

for all $s_i \in S_i$ and $i = 1, ..., I$
• Is Nash Equilibrium unique?

• Does it always exist?

• Penalty kick in soccer (matching pennies)

<table>
<thead>
<tr>
<th>Kicker \ Goalie</th>
<th>L</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>0, 1</td>
<td>1, 0</td>
</tr>
<tr>
<td>R</td>
<td>1, 0</td>
<td>0, 1</td>
</tr>
</tbody>
</table>

• Equilibrium always exists in mixed strategies $\sigma$
• Mixed strategy: allow for probability distribution.

• Back to penalty kick:
  
  – Kicker kicks left with probability $k$
  
  – Goalie kicks left with probability $g$

  – utility for kicker of playing $L$:

    $$U_K(L, \sigma) = gU_K(L, L) + (1 - g)U_K(L, R) = (1 - g)$$

  – utility for kicker of playing $R$:

    $$U_K(R, \sigma) = gU_K(R, L) + (1 - g)U_K(R, R) = g$$
• Optimum?

- $L \succ R$ if $1 - g > g$ or $g < 1/2$
- $R \succ L$ if $1 - g < g$ or $g > 1/2$
- $L \sim R$ if $1 - g = g$ or $g = 1/2$

• Plot best response for kicker

• Plot best response for goalie
• Nash Equilibrium is:
  
  – fixed point of best response correspondence

  – crossing of best response correspondences
2 Oligopoly: Cournot

- Nicholson, p. 531.

- Back to oligopoly maximization problem

- Assume 2 firms, cost $c_i(y_i) = c y_i$, $i = 1, 2$

- Firms choose simultaneously quantity $y_i$

- Firm $i$ maximizes:

\[
\max_{y_i} p (y_i + y_{-i}) y_i - c y_i.
\]

- First order condition with respect to $y_i$:

\[
p'_Y (y_i^* + y_{-i}^*) y_i^* + p - c = 0, \quad i = 1, 2.
\]
• Nash equilibrium:

- $y_1$ optimal given $y_2$;
- $y_2$ optimal given $y_1$.

• Solve equations:

$$p_Y (y_1^* + y_2^*) y_1^* + p - c = 0$$ and

$$p_Y (y_2^* + y_1^*) y_2^* + p - c = 0.$$

• Pricing above marginal cost


3 Oligopoly: Bertrand

• Previously, we assumed firms choose quantities

• Now, assume firms first choose prices, and then produce quantity demanded by market

• 2 firms

• Profits:

\[ \pi_i (p_i, p_{-i}) = \begin{cases} 
(p_i - c) Y (p_i) & \text{if } p_i < p_{-i} \\
(p_i - c) Y (p_i) / 2 & \text{if } p_i = p_{-i} \\
0 & \text{if } p_i > p_{-i}
\end{cases} \]
• First show that $p_1 = c = p_2$ is Nash Equilibrium

• Does any firm have a (strict) incentive to deviate?
• Show that this equilibrium is unique

• Case 1. \( p_1 > p_2 > c \)

• Case 2. \( p_1 = p_2 > c \)

• Case 3. \( p_1 > c \geq p_2 \)

• Case 4. \( c > p_1 \geq p_2 \)
• Case 5. $p_1 = c > p_2$

• Case 6. $p_1 = c = p_2$

• It is unique!
• Marginal cost pricing

• Two firms are enough to guarantee perfect competition!

• Price wars
4 Next lecture

- Dynamic games
- Stackelberg duopoly
- Auctions