Outline

1. Second-price Auction

2. Dynamic Games
1 Second-price Auction

• Sealed-bid auction

• Highest bidder wins object

• Price paid is second highest price

• Two individuals: $I = 2$

• Strategy $s_i$ is bid $b_i$

• Each individual knows value $v_i$
• Payoff for individual $i$ is

$$u_i(b_i, b_{-i}) = \begin{cases} 
  v_i - b_i & \text{if } b_i > b_{-i} \\
  (v_i - b_{-i}) / 2 & \text{if } b_i = b_{-i} \\
  0 & \text{if } b_i < b_{-i}
\end{cases}$$

• Show: weakly dominant to set $b^*_i = v_i$

• To show:

$$u_i(v_i, b_{-i}) \geq u_i(b_i, b_{-i})$$

for all $b_i$, for all $b_{-i}$, and for $i = 1, 2$. 
1. Assume $b_{-i} > v_i$

- $u_i(v_i, b_{-i}) = 0 = u_i(b_i, b_{-i})$ for any $b_i < b_{-i}$ [REVISED]

- $u_i(b_{-i}, b_{-i}) = (v_i - b_{-i})/2 < 0$ [REVISED]

- $u_i(b_i, b_{-i}) = (b_i - b_{-i}) < 0$ for any $b_i > b_{-i}$ [REVISED]

2. Assume now $b_{-i} = v_i$
3. Assume now \( b_i < v_i \).
2 Dynamic Games

- Nicholson, Ch. 10, pp. 256–259.

- Dynamic games: one player plays after the other

- Decision trees
  - Decision nodes
  - Strategy is a plan of action at each decision node
- Example: battle of the sexes game

<table>
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<tr>
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<th>She \ He</th>
<th>Ballet</th>
<th>Football</th>
</tr>
</thead>
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<td>2, 1</td>
<td>0, 0</td>
<td></td>
</tr>
<tr>
<td>Football</td>
<td>0, 0</td>
<td>1, 2</td>
<td></td>
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</tbody>
</table>

- Dynamic version: she plays first
• **Subgame-perfect equilibrium.** At each node of the tree, the player chooses the strategy with the highest payoff, given the other players’ strategy

• Backward induction. Find optimal action in last period and then work backward

• Solution
• Example 2: Entry Game

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<tbody>
<tr>
<td>Enter</td>
<td>$-1, -1$</td>
<td>$10, 0$</td>
</tr>
<tr>
<td>Do not Enter</td>
<td>$0, 5$</td>
<td>$0, 0$</td>
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</tbody>
</table>

• Exercise. Dynamic version.

• Coordination games solved if one player plays first.
• Can use this to study finitely repeated games

• Suppose we play the prisoner's dilemma game ten times.

\[
\begin{array}{c|ccc}
1 \setminus 2 & D & ND \\
\hline
D & -4, -4 & -1, -5 \\
ND & -5, -1 & -2, -2 \\
\end{array}
\]

• What is the subgame perfect equilibrium?