This Problem set tests the knowledge that you accumulated mainly in lectures 24 to 26. The problem set is focused on dynamic games and general equilibrium. General rules for problem sets: show your work, write down the steps that you use to get a solution (no credit for right solutions without explanation), write legibly. If you cannot solve a problem fully, write down a partial solution. We give partial credit for partial solutions that are correct. Do not forget to write your name on the problem set!

Problem 1. Dynamic Games (48 points) Two companies produce the same good. In the first period, firm 1 sells its product as a monopolist on the West Coast. In the second period, firm 1 competes with firm 2 on the East Coast as a Cournot duopolist. There is no discounting between the two periods. Firm 1 produces quantity \( x_W \) at the West at cost \( cx_W \). On the East Coast, and that’s what makes this problem interesting, firm 1 produces quantity \( x_E \) at cost \( (c - \alpha x_W) x_E \), where \( 0 < \alpha < c < 1/2 \). The parameter \( \alpha \) captures a form of learning by doing. The more firm 1 produces on the West Coast, the lower the marginal costs are going to be on the East Coast. As for firm 2, it produces in the East market with cost \( cx_2 \). The inverse demand functions are \( p_W(x_W) = 1 - x_W \) and \( p_E(x_E, x_2) = 1 - x_E - x_2 \). Each firm maximizes profit. In particular, firm 1 maximizes the total profits from its West and East coast operations.

1. Consider first the case of simultaneous choice. Assume that firm 2 does not observe \( x_W \) before making its production decision. This means that, although formally firm 1 chooses \( x_W \) first, that you should analyze the game as a simultaneous game between firm 1 and firm 2. Use Nash Equilibrium. Write down the profit function that firm 1 maximizes (careful here) and the profit function that firm 2 maximizes (5 points)

2. Write down the first order conditions of firm 1 with respect to \( x_W \) and \( x_E \), and the first order condition of firm 2 with respect to \( x_2 \). Solve for \( x_W^* \), \( x_E^* \), and \( x_2^* \). (4 points)

3. Check the second order conditions for firm 1 and for firm 2. (3 points)

4. What is the comparative statics of \( x_W^* \) and \( x_E^* \) with respect to \( \alpha \)? Does it make sense? How about the comparative statics of \( x_2^* \) with respect to \( \alpha \)? (4 points)

5. Compute the profits of firm 2 in equilibrium. How do they vary as \( \alpha \) varies? (compute the comparative statics) Why are firm 2’s profits affected by \( \alpha \) even though the parameter \( \alpha \) does not directly affect the costs of firm 2? (5 points)

6. Now consider the case of sequential choice. Assume that firm 2 observes \( x_W \) before making its production decision \( x_2 \). This means that you should analyze the game as a dynamic game between firm 1 and firm 2, and use the concept of subgame-perfect equilibrium. Remember, we start from the last period. Write down the profit functions that firm 1 and firm 2 maximize on the East Coast (4 points)

7. Write down the first order conditions of firm 1 with respect to \( x_E \), and the first order condition of firm 2 with respect to \( x_2 \). Solve for \( x_E^* \) and \( x_2^* \) as a function of \( x_W^* \). (4 points)

8. Compute the comparative statics of \( x_E^* \) and \( x_2^* \) with respect to \( x_W^* \). Do these results make sense? (3 points)

9. Compute the profits of firm 1 on the East Coast as a function of \( x_W^* \). (2 points)

10. Using the answer to point 9, write down the maximization problem of firm 1 in the first period, that it, when it decides the production on the West Coast. (3 points)

11. Write down the first order conditions of firm 1 with respect to \( x_W \). Solve for \( x_W^* \), find the solution for \( x_E^* \) and \( x_2^* \). (5 points)
12. Compare the solutions for $x_i^*$ under simultaneous and under sequential choice. What can you conclude? Under which conditions the firm does more preemption, that is, produces more on the West Coast in order to reduce the production in equilibrium of firm 2? (6 points)

**Problem 2. General Equilibrium** (25 points) Consider the case of pure exchange with two consumers. Both consumers have Cobb-Douglas preferences, but with different parameters. Consumer 1 has utility function $u(x_1^1, x_2^1) = (x_1^1)^\alpha (x_2^1)^{1-\alpha}$. Consumer 2 has utility function $u(x_1^2, x_2^2) = (x_1^2)^\beta (x_2^2)^{1-\beta}$. The endowment of good $j$ owned by consumer $i$ is $\omega_i^j$. The price of good 1 is $p_1$, while the price of good 2 is normalized to 1 without loss of generality.

1. Only for point 1, assume $\omega_1^1 = 1, \omega_2^1 = 3, \omega_1^2 = 3, \omega_2^2 = 1$. (that is, total endowment of each good is 4). Assume further $\alpha = 1/2, \beta = 1/2$. Draw the Pareto set and the contract curve for this economy in an Edgeworth box. (you do not need to give the exact solutions, only a graphical representation) What is the set of points that could be the outcome under barter in this economy? (5 points)

2. For each consumer, compute the utility maximization problem. Solve for $x_i^j$ for $j = 1, 2$ and $i = 1, 2$ as a function of the price $p_1$ and of the endowments. (By now, this part should be sooo familiar to you, you should be able to solve this problem with closed eyes) (5 points)

3. Now comes the general equilibrium part. Require now that the total sum of the demands for good 1 equals the total sum of the endowments, that is, that $x_1^1 + x_1^2 = \omega_1^1 + \omega_1^2$. Solve for the general equilibrium price $p_1^*$. (6 points)

4. What is the comparative statics of $p_1^*$ with respect to the endowment of good 1, that is, with respect to $\omega_i^1$ for $i = 1, 2$? What about with respect to the endowment of the other good? Does this make sense? What is the comparative statics of $p_1^*$ with respect to the taste for good 1, that is, with respect to $\alpha$ and $\beta$? Does this make sense? (4 points)

5. Now require the same general equilibrium condition in market 2. Solve for $p_1^*$ again, and check that this solution is the same as the one you found in the point above. In other words, you found a property that is called Walras’ Law. In an economy with $n$ markets, if $n-1$ markets are in equilibrium, the $n$th market will be in equilibrium as well. (5 points)