Economics 101A
(Lecture 11)

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October 7, 2004
Outline

1. Mid-Term Suggestions
2. Charitable Donations II
3. Risk Aversion
4. Insurance
5. Investment in Risky Asset
6. Measures of Risk Aversion
1 Mid-Term Suggestions

• Suggestions from you...
2 Charitable Donations II

• A quick look at the evidence

• From Andreoni (2002)
3 Risk aversion

• Nicholson, Ch. 18, pp. 535–541 [OLD: Ch. 8, pp. 200-206].

• Risk aversion:
  – individuals dislike uncertainty
  – $u$ concave, $u'' < 0$

• Implications?
  – purchase of insurance (possible accident)
  – investment in risky asset (risky investment)
  – choice over time (future income uncertain)
• Experiment — Are you risk-averse?
4 Insurance

• Nicholson, Ch. 18, pp. 545–551 [OLD: Ch. 8, pp. 211-216] Notice: different treatment than in class

• Individual has:
  
  – wealth \( w \)

  – utility function \( u \), with \( u' > 0, u'' < 0 \)

• Probability \( p \) of accident with loss \( L \)

• Insurance offers coverage:
  
  – premium \( q \) for each \( $1 \) paid in case of accident

  – units of coverage purchased \( \alpha \)
• Individual maximization:

$$\max_{\alpha} (1 - p) u (w - q\alpha) + pu (w - q\alpha - L + \alpha)$$

s.t. $\alpha \geq 0$

• Assume $\alpha^* \geq 0$, check later

• First order conditions:

$$0 = -q (1 - p) u' (w - q\alpha) + (1 - q) pu' (w - q\alpha - L + \alpha)$$

or

$$\frac{u' (w - q\alpha)}{u' (w - q\alpha - L + \alpha)} = \frac{1 - q}{q} \frac{p}{1 - p}.$$ 

• Assume first $q = p$ (insurance is fair)

• Solution for $\alpha^* =$?
• $\alpha^* > 0$, so we are ok!

• What if $q > p$ (insurance needs to cover operating costs)?

• Insurance will be only partial (if at all)

• Exercise: Check second order conditions!
5 Investment in Risk Asset

• Individual has:
  – wealth \( w \)
  – utility function \( u \), with \( u' > 0 \)

• Two possible investments:
  – Asset B (bond) yields return 1 for each dollar
  – Asset S (stock) yields uncertain return \((1 + r)\):
    * \( r = r_+ > 0 \) with probability \( p \)
    * \( r = r_- < 0 \) with probability \( 1 - p \)
    * \( Er = pr_+ + (1 - p) r_- > 0 \)

• Share of wealth invested in stock \( S = \alpha \)
• Individual maximization:
\[
\max_{\alpha} (1 - p) u (w [(1 - \alpha) + \alpha (1 + r_-)]) + 
+ pu (w [(1 - \alpha) + \alpha (1 + r_+)]) 
\]
\[
s.t. 0 \leq \alpha \leq 1
\]

• Case of risk neutrality: \( u(x) = a + bx, \ b > 0 \)

• Assume \( a = 0 \) (no loss of generality)

• Maximization becomes
\[
\max_{\alpha} b (1 - p) (w [1 + \alpha r_-]) + bp (w [1 + \alpha r_+]) 
\]
or
\[
\max_{\alpha} bw + \alpha bw [(1 - p) r_- + pr_+] 
\]

• Sign of term in square brackets? Positive!

• Set \( \alpha^* = 1 \)
• Case of risk aversion: \( u'' < 0 \)

• Assume \( 0 \leq \alpha^* \leq 1 \), check later

• First order conditions:

\[
0 = (1 - p)(wr_-)u'(w[1 + \alpha r_-]) +
+ p(wr_+)u'(w[1 + \alpha r_+])
\]

• Can \( \alpha^* = 0 \) be solution?

• Solution is \( \alpha^* > 0 \) (positive investment in stock)

• Exercise: Check s.o.c.
6 Next lecture and beyond

• Tu:
  – Time consistency
  – Time inconsistency
  – Application to health clubs

• Th:
  – Production!
  – Returns to scale
  – Cost minimization