Outline

1. Time Inconsistency II
2. Health Club Attendance
3. Production: Introduction
4. Production Function
5. Returns to Scale
6. Two-step Cost Minimization
1 Time Inconsistency II

- Alternative specification (Akerlof, 1991; Laibson, 1997; O’Donoghue and Rabin, 1999)

- Utility at time $t$ is $u(c_t, c_{t+1}, c_{t+2})$:

  $$u(c_t) + \frac{\beta}{1 + \delta} u(c_{t+1}) + \frac{\beta}{(1 + \delta)^2} u(c_{t+2}) + ...$$

- Discount factor is

  $$1, \frac{\beta}{1 + \delta}, \frac{\beta}{(1 + \delta)^2}, \frac{\beta}{(1 + \delta)^3}, ...$$

  instead of

  $$1, \frac{1}{1 + \delta}, \frac{1}{1 + \delta}, \frac{1}{(1 + \delta)^2}, \frac{1}{(1 + \delta)^3}, ...$$

- What is the difference?

- Immediate gratification: $\beta < 1$
Back to our problem: **Period 1.**

Maximization problem:

\[
\max U(c_1) + \frac{\beta}{1 + \delta} EU(c_2) \\
s.t. \ c_1 + \frac{1}{1 + r} c_2 \leq M_1' + \frac{1}{1 + r} M_2
\]

First order conditions:

Ratio of f.o.c.s:

\[
\frac{U'(c_1^*)}{EU'(c_2^*)} = \beta \frac{1 + r}{1 + \delta}
\]
• Now, **period 0** with commitment.

• Maximization problem:

$$\max U(c_0) + \frac{\beta}{1 + \delta} U(c_1) + \frac{\beta}{(1 + \delta)^2} EU(c_2)$$

subject to

$$c_1 + \frac{1}{1 + r}c_2 \leq M_1' + \frac{1}{1 + r}M_2$$

• First order conditions:

• Ratio of f.o.c.s:

$$\frac{U'(c_1^*, c)}{EU'(c_2^*, c)} = \frac{1 + r}{1 + \delta}$$

• The two conditions differ!

• Time inconsistency: $c_{1^*, c} < c_1^*$ and $c_{2^*, c} > c_2^*$

• The agent allows him/herself too much immediate consumption and saves too little
• Ok, we agree. but should we study this as economists?

• YES!
  – One trillion dollars in credit card debt;
  – Most debt is in teaser rates;
  – Two thirds of Americans are overweight or obese;
  – $10bn health-club industry

• Is this testable?
  – In the laboratory?
  – In the field?
2 Health Club Attendance

• Health club industry study (DellaVigna and Malmendier, 2002)

• 3 health clubs

• Data on attendance from swiping cards

• Choice of contracts:
  
  – Monthly contract with average price of $75
  
  – 10-visit pass for $100

• Consider users that choose monthly contract. Attendance?
• Attend on average 4.8 times per month

• Pay on average over $17

• Average delay of 2.2 months ($185) between last attendance and contract termination

• Over membership, user could have saved $700 by paying per visit
• Health club attendance:
  
  – immediate cost $c$
  
  – delayed benefit $b$

• At sign-up (attend tomorrow):

$$NB^t = -\frac{\beta}{1 + \delta}c + \frac{\beta}{(1 + \delta)^2}b$$

• Plan to attend if $NB^t > 0$

$$c < \frac{1}{(1 + \delta)}b$$
• Once moment to attend comes:

\[ NB = -c + \frac{\beta}{(1 + \delta)} b \]

• Attend if \( NB > 0 \)

\[ c < \frac{\beta}{(1 + \delta)} b \]
• Interpretations?

• Users are buying a commitment device

• User underestimate their future self-control problems:
  – They overestimate future attendance
  – They delay cancellation
3 Production: Introduction

- Second half of the economy. Production

- Example. Ford and the Minivan (Petrin, 2002):
  - Ford had idea: "Mini/Max" (early '70s)
  - Did Ford produce it?
    - No!
  - Ford was worried of cannibalizing station wagon sector
  - Chrysler introduces Dodge Caravan (1984)
  - Chrysler: $1.5bn profits (by 1987)!
• Why need separate treatment?

• Perhaps firms maximize utility...

• ...we can be more precise:
  – Competition
  – Institutional structure
Production Function


- Production function: \( y = f (z) \). Function \( f : \mathbb{R}^n_+ \rightarrow \mathbb{R}_+ \)

- Inputs \( z = (z_1, z_2, ..., z_n) \): labor, capital, land, human capital

- Output \( y \): Minivan, Intel Pentium III, mangoes (Philippines)

- Properties of \( f \):
  - no free lunches: \( f (0) = 0 \)
  - positive marginal productivity: \( f_1' (z) > 0 \)
  - decreasing marginal productivity: \( f'_{i,i} (z) < 0 \)
- Isoquants $Q(y) = \{x | f(x) = y\}$

- Set of inputs $z$ required to produce quantity $y$

- Special case. Two inputs:
  
  - $z_1 = L$ (labor)
  
  - $z_2 = K$ (capital)

- Isoquant: $f(L, K) - y = 0$

- Slope of isoquant $dK/dL = MRTS$
• Convex production function if convex isoquants

• Reasonable: combine two technologies and do better!

• Mathematically, \( \frac{d^2K}{d^2L} = \)
5 Returns to Scale

- Nicholson, Ch. 7, pp. 190–193 [OLD: Ch. 11, pp. 275–278]

- Effect of increase in labor: $f'_L$

- Increase of all inputs: $f(tz)$ with $t$ scalar, $t > 1$

- How much does input increase?
  
  - Decreasing returns to scale: for all $z$ and $t > 1$,
    
    $$f(tz) < tf(z)$$

  - Constant returns to scale: for all $z$ and $t > 1$,
    
    $$f(tz) = tf(z)$$
– Increasing returns to scale: for all $z$ and $t > 1$,

\[ f(tz) > tf(z) \]
• Example: \( y = f(K, L) = AK^\alpha L^\beta \)

• Marginal product of labor: \( f'_L = \)

• Decreasing marginal product of labor: \( f''_L = \)

• \textit{MRTS} =

• Convex isoquant?

• Returns to scale: \( f(tK, tL) = A(tK)^\alpha (tL)^\beta = t^{\alpha+\beta} AK^\alpha L^\beta = t^{\alpha+\beta} f(K, L) \)
6 Two-step Cost minimization

- Nicholson, pp. 212–220 [OLD, Ch. 12, pp. 298–307]

- Objective of firm: Produce output that generates maximal profit.

- Decompose problem in two:
  - Given production level $y$, choose cost-minimizing combinations of inputs
  - Choose optimal level of $y$.

- First step. Cost-Minimizing choice of inputs
Two-input case: Labor, Capital

Input prices:

- Wage $w$ is price of $L$
- Interest rate $r$ is rental price of capital $K$

Expenditure on inputs: $wL + rK$

Firm objective function:

$$\min wL + rK$$

$$s.t. f(L, K) \geq y$$
• Compare with expenditure minimization for consumers

• First order conditions:

\[ w - \lambda f'_{L} = 0 \]

and

\[ r - \lambda f'_{K} = 0 \]

• Rewrite as

\[ \frac{f'_{L}(L^*, K^*)}{f'_{K}(L^*, K^*)} = \frac{w}{r} \]

• MRTS (slope of isoquant) equals ratio of input prices
• Graphical interpretation
• Derived demand for inputs:

\[- L = L^* (w, r, y)\]

\[- K = K^* (w, r, y)\]

• Value function at optimum is \textbf{cost function:}

\[ c (w, r, y) = wL^* (r, w, y) + rK^* (r, w, y) \]
• Second step. Given cost function, choose optimal quantity of $y$ as well

• Price of output is $p$.

• Firm’s objective:

$$\max p y - c (w, r, y)$$

• First order condition:

$$p - c_y' (w, r, y) = 0$$

• Price equals marginal cost – very important!
7 Next Lecture

- Continue Cost Minimization
- Solve an Example
- Cases in which s.o.c. are not satisfied
- Start Profit Maximization