Outline

1. Cost Curves and Supply Function

2. One-step Profit Maximization

3. Introduction to Market Equilibrium

4. Aggregation

5. Market Equilibrium in the Short-Run
1 Cost Curves

- Nicholson, Ch. 8, pp. 220–228; Ch. 9, pp. 256–259 [OLD: Ch. 12, pp. 307–312 and Ch. 13, pp. 342–346.]

- Marginal costs $MC = \frac{\partial c}{\partial y} \rightarrow$ Cost minimization

  \[ p = MC = \frac{\partial c(w, r, y)}{\partial y} \]

- Average costs $AC = \frac{c}{y} \rightarrow$ Does firm break even?

  \[ \pi = py - c(w, r, y) > 0 \text{ iff} \]

  \[ \frac{\pi}{y} = p - c(w, r, y) / y > 0 \text{ iff} \]

  \[ \frac{c(w, r, y)}{y} = AC < p \]

- **Supply function.** Portion of marginal cost $MC$ above average costs. (price equals marginal cost)
• Assume only 1 input (expenditure minimization is trivial)

• **Case 1.** Production function. $y = L^\alpha$

  - Cost function? (cost of input is $w$):
    $$ c(w, y) = wL^*(w, y) = wy^{1/\alpha} $$

  - Marginal cost?
    $$ \frac{\partial c(w, y)}{\partial y} = \frac{1}{\alpha} wy^{(1-\alpha)/\alpha} $$

  - Average cost $c(w, y) / y$?
    $$ \frac{c(w, y)}{y} = \frac{wy^{1/\alpha}}{y} = wy^{(1-\alpha)/\alpha} $$
• **Case 1a.** \( \alpha > 1 \). Plot production function, total cost, average and marginal. Supply function?

• **Case 1b.** \( \alpha = 1 \). Plot production function, total cost, average and marginal. Supply function?

• **Case 1c.** \( \alpha < 1 \). Plot production function, total cost, average and marginal. Supply function?
• **Case 2.** *Non-convex technology.* Plot production function, total cost, average and marginal. Supply function?

• **Case 3.** *Technology with setup cost.* Plot production function, total cost, average and marginal. Supply function?
1.1 Supply Function

- Supply function: \( y^* = y^* (w, r, p) \)

- What happens to \( y^* \) as \( p \) increases?

- Is the supply function upward sloping?

- Remember f.o.c:
  \[
  p - c'_y (w, r, y) = 0
  \]

- Implicit function:
  \[
  \frac{\partial y^*}{\partial p} = -\frac{1}{-c''_{y,y} (w, r, y)} > 0
  \]
  as long as s.o.c. is satisfied.

- Yes! Supply function is upward sloping.
2 One-step Profit Maximization

- Nicholson, Ch. 9, pp. 265–270 [OLD: Ch. 13, pp. 346–350].

- One-step procedure: maximize profits

- Perfect competition. Price $p$ is given
  
  - Firms are small relative to market
  
  - Firms do not affect market price $p_M$

  - Will firm produce at $p > p_M$?
  
  - Will firm produce at $p < p_M$?

  $\Rightarrow p = p_M$
• Revenue: $py = pf(L, K)$

• Cost: $wL + rK$

• Profit $pf(L, K) - wL - rK$
• Agent optimization:

\[
\max_{L,K} pf(L, K) - wL - rK
\]

• First order conditions:

\[
\frac{pf_L(L, K)}{L} - w = 0
\]

and

\[
\frac{pf_K(L, K)}{K} - r = 0
\]

• Second order conditions?  \( pf''_{L,L}(L, K) < 0 \) and

\[
|H| = \begin{vmatrix} pf''_{L,L}(L, K) & pf''_{L,K}(L, K) \\ pf''_{L,K}(L, K) & pf''_{K,K}(L, K) \end{vmatrix} = p^2 \left[ f''_{L,L} f''_{K,K} - (f''_{L,K})^2 \right] > 0
\]

• Need \( f''_{L,K} \) not too large for maximum
Comparative statics with respect to $p$, $w$, and $r$.

What happens if $w$ increases?

\[
\frac{\partial L^*}{\partial w} = -\frac{\begin{vmatrix} -1 & p_f^{II}_{L,K}(L,K) \\ 0 & p_f^{II}_{K,K}(L,K) \end{vmatrix}}{\begin{vmatrix} p_f^{II}_{L,L}(L,K) & p_f^{II}_{L,K}(L,K) \\ p_f^{II}_{L,K}(L,K) & p_f^{II}_{K,K}(L,K) \end{vmatrix}} < 0
\]

and

\[
\frac{\partial L^*}{\partial r} =
\]

Sign of $\partial L^*/\partial r$ depends on $f''_{L,K}$. 
3 Introduction to Market Equilibrium

• Nicholson, Ch. 10, pp. 279–295 [OLD: Ch. 14, pp. 368–382.

• Two ways to analyze firm behavior:
  – Two-Step Cost Minimization
  – One-Step Profit Maximization

• What did we learn?
  – Optimal demand for inputs $L^*$, $K^*$ (see above)
  – Optimal quantity produced $y^*$
• **Supply function.** $y = y^* (p, w, r)$
  
  – From profit maximization:
  $$y = f \left( L^* (p, w, r), K^* (p, w, r) \right)$$
  
  – From cost minimization:
  $$MC \text{ curve above } AC$$
  
  – Supply function is increasing in $p$

• Market Equilibrium. Equate demand and supply.

• Aggregation?

• Industry supply function!
4 Aggregation

4.1 Producers aggregation

- $J$ companies, $j = 1, \ldots, J$, producing good $i$

- Company $j$ has supply function
  \[ y_i^j = y_i^{j*}(p_i, w, r) \]

- Industry supply function:
  \[ Y_i(p_i, w, r) = \sum_{j=1}^{J} y_i^{j*}(p_i, w, r) \]

- Graphically,
4.2 Consumer aggregation

- Nicholson, Ch. 10, pp. 279–282 [OLD: Ch. 7, pp. 172–176]

- One-consumer economy

- Utility function $u(x_1, \ldots, x_n)$

- Prices $p_1, \ldots, p_n$

- Maximization $\Rightarrow$

\[
\begin{align*}
x_1^* &= x_1^* (p_1, \ldots, p_n, M), \\
& \quad \vdots \\
x_n^* &= x_n^* (p_1, \ldots, p_n, M).
\end{align*}
\]
Focus on good $i$. Fix prices $p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n$ and $M$.

**Single-consumer demand function**:

\[ x_i^* = x_i^*(p_i|p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n, M) \]

- What is sign of $\partial x_i^*/\partial p_i$?
  - Negative if good $i$ is normal
  - Negative or positive if good $i$ is inferior
- **Aggregation**: $J$ consumers, $j = 1, \ldots, J$

- Demand for good $i$ by consumer $j$:
  
  $$x_{i}^{j*} = x_{i}^{j*}(p_1, \ldots, p_n, M_j)$$

- Market demand $X_i$:
  
  $$X_i (p_1, \ldots, p_n, M^1, \ldots, M^J) = \sum_{j=1}^{J} x_{i}^{j*}(p_1, \ldots, p_n, M^j)$$

- Graphically,
• Notice: market demand function depends on distribution of income $M^J$

• Market demand function $X_i$:
  – Consumption of good $i$ as function of prices $p$
  – Consumption of good $i$ as function of income distribution $M^j$
5 Market Equilibrium in the Short-Run

- Nicholson, Ch. 14, pp. 368–382.

- What is equilibrium price \( p_i \)?

- Magic of the Market...

- Equilibrium: No excess supply, No excess demand

- Prices \( p^* \) equates demand and supply of good \( i \):

\[
Y^* = Y^S_i (p^*_i, w, r) = X^D_i (p^*_1, ..., p^*_n, M^1, ..., M^J)
\]
• Graphically,

• Notice: in short-run firms can make positive profits
• Comparative statics exercises with endogenous price $p_i$:
  
  – increase in wage $w$ or interest rate $r$:

  – change in income distribution
6 Next Lecture

- Comparative Statics of Equilibrium
- Taxes and Subsidies
- Long-Run Equilibrium