Outline

1. Oligopoly?

2. Game Theory

3. Oligopoly: Cournot

4. Oligopoly: Bertrand
1 Oligopoly?

• Extremes:
  – Perfect competition
  – Monopoly

• Oligopoly if there are $n$ (two, five...) firms

• Examples:
  – soft drinks: Coke, Pepsi;
  – cellular phones: Sprint, AT&T, Cingular,...
  – car dealers
• Firm $i$ maximizes:

$$\max_{y_i} p \left(y_i + y_{-i}\right) y_i - c(y_i)$$

where $y_{-i} = \sum_{j \neq i} y_j$.

• First order condition with respect to $y_i$:

$$p_Y' \left(y_i + y_{-i}\right) y_i + p - c_y'(y_i) = 0.$$

• Problem: what is the value of $y_{-i}$?

  – simultaneous determination?

  – can firms $-i$ observe $y_i$?

• Need to study strategic interaction
2 Game Theory

- Nicholson, Ch. 15, pp. 440–449 [OLD: Ch. 10, pp. 246–255].

- Unfortunate name

- Game theory: study of decisions when payoff of player \( i \) depends on actions of player \( j \).

- Brief history:
  - von Neuman and Morgenstern, Theory of Games and Economic Behavior (1944)
  - Nash, Non-cooperative Games (1951)
  - ...
  - Nobel Prize to Nash, Harsanyi (Berkeley), Selten (1994)
• Definitions:

  – Players: $1, \ldots, I$

  – Strategy $s_i \in S_i$

  – Payoffs: $U_i (s_i, s_{-i})$
- Example: Prisoner’s Dilemma

  - \( I = 2 \)

  - \( s_i = \{D, ND\} \)

  - Payoffs matrix:

\[
\begin{array}{ccc}
1 \mid 2 & D & ND \\
D & -4, -4 & -1, -5 \\
ND & -5, -1 & -2, -2 \\
\end{array}
\]
• What prediction?

• Maximize sum of payoffs

• Choose dominant strategies

• **Equilibrium in dominant strategies**

• Strategies \( s^* = (s^*_i, s^*_{-i}) \) are an Equilibrium in dominant strategies if

\[
U_i (s^*_i, s^*_{-i}) \geq U_i (s_i, s_{-i})
\]

for all \( s_i \in S_i \), for all \( s_{-i} \in S_{-i} \) and all \( i = 1, \ldots, I \)
• Battle of the Sexes game:

<table>
<thead>
<tr>
<th></th>
<th>Ballet</th>
<th>Football</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ballet</td>
<td>2, 1</td>
<td>0, 0</td>
</tr>
<tr>
<td>Football</td>
<td>0, 0</td>
<td>1, 2</td>
</tr>
</tbody>
</table>

• Choose dominant strategies? Not possible

• Nash Equilibrium.

• Strategies \( s^* = (s^*_i, s^*_{-i}) \) are a Nash Equilibrium if

\[
U_i \left( s^*_i, s^*_{-i} \right) \geq U_i \left( s_i, s^*_{-i} \right)
\]

for all \( s_i \in S_i \) and \( i = 1, \ldots, I \)
• Is Nash Equilibrium unique?

• Does it always exist?

• Penalty kick in soccer (matching pennies)

<table>
<thead>
<tr>
<th>Kicker \ Goalie</th>
<th>L</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>0,1</td>
<td>1,0</td>
</tr>
<tr>
<td>R</td>
<td>1,0</td>
<td>0,1</td>
</tr>
</tbody>
</table>

• Equilibrium always exists in mixed strategies $\sigma$
• Mixed strategy: allow for probability distribution.

• Back to penalty kick:
  
  – Kicker kicks left with probability $k$
  
  – Goalie kicks left with probability $g$

  – utility for kicker of playing $L$:
    \[ U_{K}(L, \sigma) = gU_{K}(L, L) + (1 - g)U_{K}(L, R) \]
    \[ = (1 - g) \]

  – utility for kicker of playing $R$:
    \[ U_{K}(R, \sigma) = gU_{K}(R, L) + (1 - g)U_{K}(R, R) \]
    \[ = g \]
Optimum?

- $L \succ R$ if $1 - g > g$ or $g < 1/2$
- $R \succ L$ if $1 - g < g$ or $g > 1/2$
- $L \sim R$ if $1 - g = g$ or $g = 1/2$

- Plot best response for kicker

- Plot best response for goalie
• Nash Equilibrium is:
  – fixed point of best response correspondence
  – crossing of best response correspondences
3 Oligopoly: Cournot


- Back to oligopoly maximization problem

- Assume 2 firms, cost \( c_i(y_i) = cy_i, \ i = 1, 2 \)

- Firms choose simultaneously quantity \( y_i \)

- Firm \( i \) maximizes:

\[
\max_{y_i} p (y_i + y_{-i}) y_i - cy_i.
\]

- First order condition with respect to \( y_i \):

\[
p'_{Y} (y^*_i + y^*_{-i}) y^*_i + p - c = 0, \ i = 1, 2.
\]
• Nash equilibrium:
  
  – $y_1$ optimal given $y_2$;

  – $y_2$ optimal given $y_1$.

• Solve equations:

  $$p_Y' (y_1^* + y_2^*) y_1^* + p - c = 0 \quad \text{and} \quad p_Y' (y_2^* + y_1^*) y_2^* + p - c = 0.$$ 

• Cournot -> Pricing above marginal cost
4 Oligopoly: Bertrand

- Previously, we assumed firms choose quantities

- Now, assume firms first choose prices, and then produce quantity demanded by market

- 2 firms

- Profits:

\[
\pi_i (p_i, p_{-i}) = \begin{cases} 
(p_i - c) Y (p_i) & \text{if } p_i < p_{-i} \\
(p_i - c) Y (p_i) / 2 & \text{if } p_i = p_{-i} \\
0 & \text{if } p_i > p_{-i}
\end{cases}
\]
• First show that $p_1 = c = p_2$ is Nash Equilibrium

• Does any firm have a (strict) incentive to deviate?
• Show that this equilibrium is unique

• Case 1. $p_1 > p_2 > c$

• Case 2. $p_1 = p_2 > c$

• Case 3. $p_1 > c \geq p_2$

• Case 4. $c > p_1 \geq p_2$
• Case 5. \( p_1 = c > p_2 \)

• Case 6. \( p_1 = c = p_2 \)

• It is unique!
• Marginal cost pricing

• Two firms are enough to guarantee perfect competition!

• Price wars
5 Next lecture

- Dynamic games
- Stackelberg duopoly
- Auctions