Economics 101A
(Lecture 22)

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Outline

1. Dynamic Games

2. Oligopoly: Stackelberg

3. General Equilibrium: Introduction

4. Edgeworth Box: Pure Exchange

5. Barter
1 Dynamic Games

- Nicholson, Ch. 15, pp. 449–454.[OLD: Ch. 10, pp. 256–259]

- Dynamic games: one player plays after the other

- Decision trees
  - Decision nodes
  - Strategy is a plan of action at each decision node
• Example: battle of the sexes game

<table>
<thead>
<tr>
<th></th>
<th>She</th>
<th>He</th>
<th>Ballet</th>
<th>Football</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ballet</td>
<td>2, 1</td>
<td>0, 0</td>
<td>0, 0</td>
<td>1, 2</td>
</tr>
</tbody>
</table>

• Dynamic version: she plays first
• **Subgame-perfect equilibrium.** At each node of the tree, the player chooses the strategy with the highest payoff, given the other players’ strategy.

• Backward induction. Find optimal action in last period and then work backward.

• Solution
• Example 2: Entry Game

<table>
<thead>
<tr>
<th></th>
<th>Enter</th>
<th>Do not Enter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enter</td>
<td>$-1, -1$</td>
<td>$10, 0$</td>
</tr>
<tr>
<td>Do not Enter</td>
<td>$0, 5$</td>
<td>$0, 0$</td>
</tr>
</tbody>
</table>

• Exercise. Dynamic version.

• Coordination games solved if one player plays first
• Can use this to study finitely repeated games

• Suppose we play the prisoner's dilemma game ten times.

\[
\begin{array}{c|cc}
1 \backslash 2 & D & ND \\
\hline 
D & -4, -4 & -1, -5 \\
ND & -5, -1 & -2, -2 \\
\end{array}
\]

• What is the subgame perfect equilibrium?
• When does repetition lead to cooperation in Prisoner Dilemma?

• Need infinite repetition

• At every period probability $p$ that game will be played again

• Strategy:
  
  – Cooperate (play $ND$)

  – If someone defected (played $D$) in past, defect (play $D$) thereafter

• See Econ 104 for more on this
2 Oligopoly: Stackelberg

- Setting as in problem set.

- 2 Firms

- Cost: \( c(y) = cy \), with \( c > 0 \)

- Demand: \( p(Y) = a - bY \), with \( a > c > 0 \) and \( b > 0 \)

- Difference: Firm 1 makes the quantity decision first

- Use subgame perfect equilibrium
• Solution:

• Solve first for Firm 2 decision as function of Firm 1 decision:

\[
\max_{y_2} (a - by_2 - by_1^*) y_2 - cy_2
\]

• F.o.c.:

\[
a - 2by_2^* - by_1^* - c = 0
\]

or

\[
y_2^* = \frac{a - c}{2b} - \frac{y_1^*}{2}.
\]

\[
p_D^* = a - bY_D^* = a - b \left(2\frac{a - c}{3b}\right) = \frac{1}{3}a + \frac{2}{3}c.
\]
• Firm 1 takes this response into account in the maximization:

$$\max_{y_1} (a - by_1 - by_2^*(y_1)) y_1 - cy_1$$

or

$$\max_{y_1} \left( a - by_1 - b \left( \frac{a - c}{2b} - \frac{y_1}{2} \right) \right) y_1 - cy_1$$

• F.o.c.:

$$a - 2by_1 - \frac{(a - c)}{2} + by_1 - c = 0$$

or

$$y_1^* = \frac{a - c}{2b}$$

and

$$y_2^* = \frac{a - c}{2b} - \frac{y_1^*}{2} = \frac{a - c}{2b} - \frac{a - c}{4b} = \frac{a - c}{4b}.$$
• Total production:

\[ Y_D^* = y_1^* + y_2^* = \frac{3(a - c)}{4b} \]

• Price equals

\[ p^* = a - b \left( \frac{3a - c}{4b} \right) = \frac{1}{4}a + \frac{3}{4}c \]

• Compare to monopoly:

\[ y_M^* = \frac{a - c}{2b} \]

and

\[ p_M^* = \frac{a + c}{2} \]

• Compare to Cournot:

\[ Y_D^* = y_1^* + y_2^* = \frac{2(a - c)}{3b} \]

and

\[ p_D^* = \frac{1}{3}a + \frac{2}{3}c. \]
• Figure

• Compare with Cournot outcome
3 General Equilibrium: Introduction

• So far, we looked at consumers
  – Demand for goods
  – Choice of leisure and work
  – Choice of risky activities

• We also looked at producers:
  – Production in perfectly competitive firm
  – Production in monopoly
  – Production in oligopoly
• We also combined consumers and producers:
  – Supply
  – Demand
  – Market equilibrium

• Partial equilibrium: one good at a time

• General equilibrium: Demand and supply for all goods!
  – supply of young worker↑ \implies \text{wage of experienced workers?}
  – minimum wage↑ \implies \text{effect on higher earners?}
  – steel tariff↑ \implies \text{effect on car price}
4 Edgeworth Box: Pure Exchange

- Nicholson, Ch. 12, pp. 335–338, 369–370 [OLD: Ch. 16, pp. 422-425]

- 2 consumers in economy: $i = 1, 2$

- 2 goods, $x_1, x_2$

- Endowment of consumer $i$, good $j$: $\omega^i_j$

- Total endowment: $(\omega_1, \omega_2) = (\omega^1_1 + \omega^2_1, \omega^1_2 + \omega^2_2)$

- Draw Edgeworth box
- Draw preferences of agent 1

- Draw preferences of agent 2
• Consumption of consumer $i$, good $j$: $x^i_j$

• Feasible consumption:

$$x^1_i + x^2_i \leq \omega_i \text{ for all } i$$

• If preferences monotonic, $x^1_i + x^2_i = \omega_i \text{ for all } i$

• Can map consumption levels into box
5 Barter

- Consumers can trade goods 1 and 2

- Allocation \( ((x_1^1, x_2^1), (x_1^2, x_2^2)) \) can be outcome of barter if:

- **Individual rationality.**
  \[ u_i(x_1^i, x_2^i) \geq u_i(\omega_1^i, \omega_2^i) \text{ for all } i \]

- **Pareto Efficiency.** There is no allocation \( ((\hat{x}_1^1, \hat{x}_2^1), (\hat{x}_1^2, \hat{x}_2^2)) \) such that
  \[ u_i(\hat{x}_1^i, \hat{x}_2^i) \geq u_i(x_1^i, x_2^i) \text{ for all } i \]
  with strict inequality for at least one agent.
• Barter outcomes in Edgeworth box

• Endowments \((\omega_1, \omega_2)\)

• Area that satisfies individual rationality condition

• Points that satisfy pareto efficiency

• **Pareto set.** Set of points where indifference curves are tangent
• **Contract curve.** Subset of Pareto set inside the individually rational area.

• Contract curve = Set of barter equilibria

• Multiple equilibria. Depends on bargaining power.

• Bargaining is time- and information-intensive procedure

• What if there are prices instead?
6  Next lecture

- Example of General equilibrium

- General Equilibrium with Prices