Outline

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1 Stackelberg II

- Firm 2 best response function:
  \[ y_2^* = \frac{a - c}{2b} - \frac{y_1^*}{2} \]

- Firm 1 best response function:
  \[ y_1^* = \frac{a - c}{2b} - \frac{y_2^*}{2} \]

- Intersection gives Cournot
• Stackelberg: Equilibrium is point on Best Response of Firm 2 that maximizes profits of Firm 1

• Plot iso-profit curve of Firm 1:

\[ \bar{\Pi} = (a - c)y_2 - by_1y_2 - by_2^2 \]

• Solve for \( y_1 \) along iso-profit:

\[ y_1 = \frac{a - c}{b} - y_2 - \frac{\bar{\Pi}}{by_2} \]

• Iso-profit curve is flat for

\[ \frac{dy_1}{dy_2} = -1 + \frac{\bar{\Pi}}{b(y_2)^2} = 0 \]

or

\[ y_2 = \]
• Figure
2 General Equilibrium: Introduction

- So far, we looked at consumers
  - Demand for goods
  - Choice of leisure and work
  - Choice of risky activities

- We also looked at producers:
  - Production in perfectly competitive firm
  - Production in monopoly
  - Production in oligopoly
• We also combined consumers and producers:
  
  – Supply

  – Demand

  – Market equilibrium

• Partial equilibrium: one good at a time

• General equilibrium: Demand and supply for all goods!
  
  – supply of young worker↑ \Rightarrow \text{wage of experienced workers?}

  – minimum wage↑ \Rightarrow \text{effect on higher earners?}

  – steel tariff↑ \Rightarrow \text{effect on car price}
3 Edgeworth Box: Pure Exchange

- Nicholson, Ch. 12, pp. 335–338, 369–370 [OLD: Ch. 16, pp. 422-425]

- 2 consumers in economy: \( i = 1, 2 \)

- 2 goods, \( x_1, x_2 \)

- Endowment of consumer \( i \), good \( j \): \( \omega_i^j \)

- Total endowment: \((\omega_1, \omega_2) = (\omega_1^1 + \omega_1^2, \omega_2^1 + \omega_2^2)\)

- No production here. With production (as in book), \((\omega_1, \omega_2)\) are optimally produced
• Edgeworth box

• Draw preferences of agent 1

• Draw preferences of agent 2
• Consumption of consumer $i$, good $j$: $x^i_j$

• Feasible consumption:

$$x^1_i + x^2_i \leq \omega_i \text{ for all } i$$

• If preferences monotonic, $x^1_i + x^2_i = \omega_i$ for all $i$

• Can map consumption levels into box
4 Barter

- Consumers can trade goods 1 and 2

- Allocation \( (x_1^1, x_2^1), (x_1^2, x_2^2) \) can be outcome of barter if:

- **Individual rationality.**

  \[
  u_i(x_1^i, x_2^i) \geq u_i(\omega_1^i, \omega_2^i) \text{ for all } i
  \]

- **Pareto Efficiency.** There is no allocation \( \left((\hat{x}_1^1, \hat{x}_2^1), (\hat{x}_1^2, \hat{x}_2^2)\right) \) such that

  \[
  u_i(\hat{x}_1^i, \hat{x}_2^i) \geq u_i(x_1^i, x_2^i) \text{ for all } i
  \]

  with strict inequality for at least one agent.
• Barter outcomes in Edgeworth box

• Endowments \((\omega_1, \omega_2)\)

• Area that satisfies individual rationality condition

• Points that satisfy pareto efficiency

• **Pareto set.** Set of points where indifference curves are tangent
• **Contract curve.** Subset of Pareto set inside the individually rational area.

• Contract curve = Set of barter equilibria

• Multiple equilibria. Depends on bargaining power.

• Bargaining is time- and information-intensive procedure

• What if there are prices instead?
5 Walrasian Equilibrium

- Prices $p_1, p_2$

- Consumer 1 faces a budget set:

$$p_1x_1^1 + p_2x_2^1 \leq p_1\omega_1^1 + p_2\omega_2^1$$

- How about consumer 2?

- Budget set of consumer 2:

$$p_1x_1^2 + p_2x_2^2 \leq p_1\omega_1^2 + p_2\omega_2^2$$

or (assuming $x_i^1 + x_i^2 = \omega_i$)

$$p_1(\omega_1 - x_1^1) + p_2(\omega_1 - x_1^2) \leq p_1(\omega_1 - \omega_1^1) + p_2(\omega_2 - \omega_2^1)$$

or

$$p_1x_1^1 + p_2x_2^1 \geq p_1\omega_1^1 + p_2\omega_2^1$$
• **Walrasian Equilibrium.** \((x_1^1, x_1^2), (x_2^1, x_2^2), (p_1^*, p_2^*)\) is a Walrasian Equilibrium if:

- Each consumer maximizes utility subject to budget constraint:

  \[
  (x_1^{i*}, x_2^{i*}) = \arg \max_{x_1^i, x_2^i} u_i \left( (x_1^i, x_2^i) \right) \\
  \text{s.t. } p_1^* x_1^i + p_2^* x_2^i \leq p_1^* \omega_1^i + p_2^* \omega_2^i
  \]

- All markets clear:

  \[
  x_j^{1*} + x_j^{2*} \leq \omega_j^1 + \omega_j^2 \text{ for all } j.
  \]
• Compare with partial (Marshallian) equilibrium:
  – each consumer maximizes utility
  – market for good \( i \) clears.
  – (no requirement that all markets clear)

• How do we find the Walrasian Equilibria?
- **Graphical method.**

  1. Compute first for each consumer set of utility-maximizing points as function of prices

  2. Check that market-clearing condition holds

- **Step 1.** Compute optimal points as prices $p_1$ and $p_2$ vary

- Start with Consumer 1. Find points of tangency between budget sets and indifference curves

- Figure
• **Offer curve** for consumer 1:

\[ (x_1^* (p_1, p_2, (\omega_1, \omega_2)), x_2^* (p_1, p_2, (\omega_1, \omega_2))) \]

• Offer curve is set of points that maximize utility as function of prices \( p_1 \) and \( p_2 \).

• Then find offer curve for consumer 2:

\[ (x_1^* (p_1, p_2, (\omega_1, \omega_2)), x_2^* (p_1, p_2, (\omega_1, \omega_2))) \]

• Figure
• Step 2. Find intersection(s) of two offer curves

• Walrasian Equilibrium is intersection of the two offer curves!
  – Both individuals maximize utility given prices
  – Total quantity demanded equals total endowment
• Relate Walrasian Equilibrium to barter equilibrium.

• Walrasian Equilibrium is a subset of barter equilibrium:
  – Does WE satisfy Individual Rationality condition?
  – Does WE satisfy the Pareto Efficiency condition?

• Walrasian Equilibrium therefore picks one (or more) point(s) on contract curve.
6 Example

• Consumer 1 has Leontieff preferences:

\[ u(x_1, x_2) = \min \left( x_1^1, x_2^1 \right) \]

• Bundle demanded by consumer 1:

\[
\begin{align*}
x_1^{1*} &= x_2^{1*} = x^{1*} = \frac{p_1 \omega_1^1 + p_2 \omega_2^1}{p_1 + p_2} = \\
&= \frac{\omega_1^1 + (p_2/p_1) \omega_2^1}{1 + (p_2/p_1)}
\end{align*}
\]

• Notice: Only ratio of prices matters (general feature)
• Consumer 2 has Cobb-Douglas preferences:

\[ u(x_1, x_2) = (x_1^2)^{.5} (x_2^2)^{.5} \]

• Demands of consumer 2:

\[ x_1^{2*} = \frac{.5 \left( p_1 \omega_1^1 + p_2 \omega_2^1 \right)}{p_1} = .5 \left( \frac{\omega_1^1}{p_1} + \frac{p_2}{p_1} \omega_2^1 \right) \]

and

\[ x_2^{2*} = \frac{.5 \left( p_1 \omega_1^1 + p_2 \omega_2^1 \right)}{p_2} = .5 \left( \frac{p_1}{p_2} \omega_1^1 + \omega_2^1 \right) \]
• Impose Walrasian equilibrium in market 1:

\[ x_1^1* + x_2^1* = \omega_1^1 + \omega_2^1 \]

This implies

\[
\frac{\omega_1^1 + (p_2/p_1) \omega_2^1}{1 + (p_2/p_1)} + .5 \left( \frac{p_2}{p_1} \right) \omega_1^1 = \omega_1^1 + \omega_2^1
\]

or

\[
.5 - .5 \left( \frac{p_2}{p_1} \right) \omega_1^1 + .5 \left( \frac{p_2}{p_1} \right) + \omega_1^1 - \left( \frac{p_2}{p_1} \right)^2 = 0
\]

or

\[
(\omega_1^1 - 2\omega_2^1) + (\omega_1^1 + \omega_2^1) (p_2/p_1) + \omega_2^1 (p_2/p_1)^2 = 0
\]
• Solution for $p_2/p_1$:

$$\frac{p_2}{p_1} = \frac{-\left(\omega_1^1 - 2\omega_2^1\right) + \sqrt{\frac{\left(\omega_1^1 + \omega_2^1\right)^2}{-4\left(\omega_1^1 - 2\omega_2^1\right)\omega_2^1}}}{2\left(\omega_1^1 - 2\omega_2^1\right)}$$

• Some complicated solution!

• Problem set has solution that is much easier to compute (and interpret)