Outline

1. Walrasian Equilibrium

2. Example

3. Welfare Theorems

4. Existence and Uniqueness

5. Empirical Economics
1 Walrasian Equilibrium

- Prices $p_1$, $p_2$

- Consumer 1 faces a budget set:
  \[ p_1 x_1^1 + p_2 x_2^1 \leq p_1 \omega^1_1 + p_2 \omega^1_2 \]

- How about consumer 2?

- Budget set of consumer 2:
  \[ p_1 x_1^2 + p_2 x_2^2 \leq p_1 \omega^2_1 + p_2 \omega^2_2 \]
  or (assuming $x_i^1 + x_i^2 = \omega_i$)
  \[ p_1 (\omega_1 - x_1^1) + p_2 (\omega_1 - x_2^1) \leq p_1 (\omega_1 - \omega^1_1) + p_2 (\omega_2 - \omega^1_2) \]
  or
  \[ p_1 x_1^1 + p_2 x_2^1 \geq p_1 \omega^1_1 + p_2 \omega^1_2 \]
• **Walrasian Equilibrium.** \( ((x_1^1, x_2^1), (x_1^2, x_2^2), (p_1^*, p_2^*)) \) is a Walrasian Equilibrium if:

- Each consumer maximizes utility subject to budget constraint:

\[
(x_1^i, x_2^i) = \arg \max_{x_1^i, x_2^i} u_i((x_1^i, x_2^i))
\]

\[
s.t. \ p_1^* x_1^i + p_2^* x_2^i \leq p_1^* \omega_1^i + p_2^* \omega_2^i
\]

- All markets clear:

\[
x_j^1 + x_j^2 \leq \omega_j^1 + \omega_j^2 \text{ for all } j.
\]
• Compare with partial (Marshallian) equilibrium:
  – each consumer maximizes utility
  – market for good $i$ clears.
  – (no requirement that all markets clear)

• How do we find the Walrasian Equilibria?
• **Graphical method.**

1. Compute first for each consumer set of utility-maximizing points as function of prices

2. Check that market-clearing condition holds

• **Step 1.** Compute optimal points as prices $p_1$ and $p_2$ vary

• Start with Consumer 1. Find points of tangency between budget sets and indifference curves

• Figure
• **Offer curve** for consumer 1:

\[(x_1^1(p_1, p_2, (\omega_1, \omega_2)), x_2^1(p_1, p_2, (\omega_1, \omega_2)))\]

• Offer curve is set of points that maximize utility as function of prices \(p_1\) and \(p_2\).

• Then find offer curve for consumer 2:

\[(x_1^2(p_1, p_2, (\omega_1, \omega_2)), x_2^2(p_1, p_2, (\omega_1, \omega_2)))\]

• Figure
• *Step 2.* Find intersection(s) of two offer curves

• Walrasian Equilibrium is intersection of the two offer curves!
  
  – Both individuals maximize utility given prices
  
  – Total quantity demanded equals total endowment
• Relate Walrasian Equilibrium to barter equilibrium.

• Walrasian Equilibrium is a subset of barter equilibrium:
  
  – Does WE satisfy Individual Rationality condition?

  – Does WE satisfy the Pareto Efficiency condition?

• Walrasian Equilibrium therefore picks one (or more) point(s) on contract curve.
2 Example

- Consumer 1 has Leontieff preferences:
  \[ u(x_1, x_2) = \min \left( x_1^1, x_2^1 \right) \]

- Bundle demanded by consumer 1:
  \begin{align*}
  x_1^1* &= x_2^1* = x^1* = \frac{p_1 \omega_1^1 + p_2 \omega_2^1}{p_1 + p_2} \\
  &= \frac{\omega_1^1 + (p_2/p_1) \omega_2^1}{1 + (p_2/p_1)}
  \end{align*}

- Notice: Only ratio of prices matters (general feature)
Consumer 2 has Cobb-Douglas preferences:

\[ u(x_1, x_2) = (x_1^2)^{.5} (x_2^2)^{.5} \]

Demands of consumer 2:

\[
\begin{align*}
  x_1^{2*} &= \frac{.5 \left( p_1 \omega_1^1 + p_2 \omega_2^1 \right)}{p_1} = .5 \left( \omega_1^1 + \frac{p_2}{p_1} \omega_2^1 \right) \\
  x_2^{2*} &= \frac{.5 \left( p_1 \omega_1^1 + p_2 \omega_2^1 \right)}{p_2} = .5 \left( \frac{p_1}{p_2} \omega_1^1 + \omega_2^1 \right)
\end{align*}
\]
• Impose Walrasian equilibrium in market 1:

\[ x_1^1* + x_1^2* = \omega_1^1 + \omega_1^2 \]

This implies

\[
\frac{\omega_1^1 + (p_2/p_1) \omega_2^1}{1 + (p_2/p_1)} + 0.5 \left( \omega_1^1 + \frac{p_2}{p_1} \omega_2^1 \right) = \omega_1^1 + \omega_1^2
\]

or

\[
\frac{0.5 - 0.5 (p_2/p_1) \omega_1^1 + 0.5 (p_2/p_1) + 0.5 (p_2/p_1)^2 - 1}{1 + (p_2/p_1)} \omega_2^1 = 0
\]

or

\[
(\omega_1^1 - 2\omega_2^1) + (\omega_1^1 + \omega_2^1) (p_2/p_1) + \omega_2^1 (p_2/p_1)^2 = 0
\]
• Solution for $p_2/p_1$:

$$\frac{p_2}{p_1} = -\left(\omega_1^1 - 2\omega_2^1\right) + \frac{\sqrt{-4 \left(\omega_1^1 - 2\omega_2^1\right) \omega_2^1}}{2 \left(\omega_1^1 - 2\omega_2^1\right)}$$

• Some complicated solution!

• Problem set has solution that is much easier to compute (and interpret)
3 Existence, Uniqueness

• Does Walrasian Equilibrium always exist?

• Not always. Example of nonexistence with non-convexity.
• Is Walrasian Equilibrium always unique?

• Not necessarily. Counterexample.
4 Welfare Theorems

- **First Fundamental Welfare Theorem.** All Walrasian Equilibria are on Contract Curve (and therefore are Pareto Efficient).

- **Proof.** Let \((x_1^*, x_2^*), (x_1^*, x_2^*)\) be a WE. Assume by contradiction that there exists a feasible bundle \(((\hat{x}_1^1, \hat{x}_1^1), (\hat{x}_2^2, \hat{x}_2^2))\) that both agents prefer to the WE. Then either \(p\hat{x}^1 \leq p\omega^1\) or \(p\hat{x}^2 \leq p\omega^2\). This contradicts definition of WE.

- **Figure**
• **Second Fundamental Welfare theorem.** Given convex preferences, for every Pareto efficient allocation \(((x_1^1, x_1^1), (x_1^2, x_2^2))\) there exists some endowment \((\omega_1, \omega_2)\) such that \(((x_1^1, x_1^1), (x_1^2, x_2^2))\) is a Walrasian Equilibrium for endowment \((\omega_1, \omega_2)\).

• Figure
• Significance of these results:

  – First Theorem: Smithian Invisible Hand. Market leads to an allocation that is Pareto Efficient.

  – BUT: problems with externalities and public good

  – BUT: what about distribution?

  – Second Theorem: Can redistribute endowments to achieve any efficient outcome as a WE.

  – But redistribution is hard to implement, and distortive.
5 Empirical Economics

- So far we have focused on economic theory

- What have we learnt (maybe)?

- Power of models

- **Consumers.** We tried to capture:
  - savings decisions (consumer today/consumer in future)
  - work-leisure trade-off (how much to work?)
  - attitudes toward risk (insurance, investment)
  - self-control problems (health club, retirement saving)
  - altruism (charitable contribution, volunteer work)
• **Producers.**

• Beauty of competitive markets:
  
  – price equals marginal costs
  
  – zero profit with entry into market
  
  – welfare optimality (no deadweight loss)

• Market power, the realistic scenario:
  
  – choice of price to maximize profits
  
  – single price or price discrimination
  
  – interaction between oligopolists
• But this is only half of economics!

• The other half is empirical economics

• Creative and careful use of data

• Get empirical answers to questions above (and other questions)

• Next week:
  – home insurance and deductible choice
  – media bias
  – ...