Economics 101A
(Lecture 7)

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Outline

1. Comparative Statics (introduction)

2. Income changes

3. Price Changes

4. Expenditure minimization

5. Slutsky Equation: Intuition
1 Comparative Statics (introduction)


- Utility maximization yields $x_i^* = x_i^*(p_1, p_2, M)$

- Quantity consumed as a function of income and price

- What happens to quantity consumed $x_i^*$ as prices or income varies?
• Simple case: Equal increase in prices and income.

• $M' = tM$, $p_1' = tp_1$, $p_2' = tp_2$.

• Compare $x^*(tM, tp_1, tp_2)$ and $x^*(M, p_1, p_2)$.

• What happens?

• Write budget line: $tp_1 x_1 + tp_2 x_2 = tM$

• Demand is homogeneous of degree 0 in $p$ and $M$:
  \[ x^*(tM, tp_1, tp_2) = t^0 x^*(M, p_1, p_2) = x^*(M, p_1, p_2). \]
Consider Cobb-Douglas Case:

\[ x_1^* = \frac{\alpha}{\alpha + \beta} M/p_1, \quad x_2^* = \frac{\beta}{\alpha + \beta} M/p_2 \]

- What is \( \frac{\partial x^*}{\partial M} \)?

- What is \( \frac{\partial x^*}{\partial p_x} \)?

- What is \( \frac{\partial x^*}{\partial p_y} \)?

- General results?
2 Income changes

- Income increases from $M$ to $M' > M$.

- Budget line $(p_1x_1 + p_2x_2 = M)$ shifts out:

$$x_2 = \frac{M'}{p_2} - x_1 \frac{p_1}{p_2}$$

- New optimum?
• Engel curve: \( x_i^*(M) \): demand for good \( i \) as function of income \( M \) holding fixed prices \( p_1, p_2 \)

• Does \( x_i^* \) increase with \( M \)?
  - Yes. Good \( i \) is *normal*
  - No. Good \( i \) is *inferior*
3 Price changes

• Price of good \( i \) increases from \( p_i \) to \( p'_i \) \( > p_i \)

• For example, decrease in price of good 2, \( p'_2 < p_2 \)

• Budget line tilts:

\[
x_2 = \frac{M}{p'_2} - x_1 \frac{p_1}{p'_2}
\]

• New optimum?
• Demand curve: $x_i^*(p_i)$: demand for good $i$ as function of own price holding fixed $p_j$ and $M$

• Odd convention of economists: plot price $p_i$ on vertical axis and quantity $x_i$ on horizontal axis. Better get used to it!
• Does $x_i^*$ decrease with $p_i$?

  – Yes. Most cases

  – No. Good $i$ is Giffen

  – Ex.: Potatoes in Ireland

  – Do not confuse with Veblen effect for luxury goods or informational asymmetries: these effects are real, but not included in current model
4 Expenditure minimization


- Solve problem **EMIN** (minimize expenditure):
  \[
  \min p_1 x_1 + p_2 x_2 \\
  \text{s.t. } u(x_1, x_2) \geq \bar{u}
  \]

- Choose bundle that attains utility $\bar{u}$ with minimal expenditure

- Ex.: You are choosing combination CDs/restaurant to make a friend happy

- If utility $u$ strictly increasing in $x_i$, can maximize s.t. equality

- Denote by $h_i(p_1, p_2, \bar{u})$ solution to EMIN problem

- $h_i(p_1, p_2, \bar{u})$ is **Hicksian or compensated demand**
• Graphically:
  
  - Fix indifference curve at level $\bar{u}$
  
  - Consider budget sets with different $M$
  
  - Pick budget set which is tangent to indifference curve

• Optimum coincides with optimum of Utility Maximization!

• Formally:

$$h_i(p_1, p_2, \bar{u}) = x^*_i(p_1, p_2, e(p_1, p_2, \bar{u}))$$
• Expenditure function is expenditure at optimum

• \( e(p_1, p_2, \bar{u}) = p_1 h_1(p_1, p_2, \bar{u}) + p_2 h_2(p_1, p_2, \bar{u}) \)

• \( h_i(p_i) \) is *Hicksian or compensated demand* function

• Is \( h_i \) always decreasing in \( p_i \)? Yes!

• Graphical proof: moving along a convex indifference curve

• (For non-convex indifferent curves, still true)
Using first order conditions:

\[ L(x_1, x_2, \lambda) = p_1 x_1 + p_2 x_2 - \lambda (u(x_1, x_2) - \bar{u}) \]

\[ \frac{\partial L}{\partial x_i} = p_i - \lambda u'_i(x_1, x_2) = 0 \]

Write as ratios:

\[ \frac{u'_1(x_1, x_2)}{u'_2(x_1, x_2)} = \frac{p_1}{p_2} \]

\[ MRS = \text{ratio of prices as in utility maximization!} \]

However: different constraint \(\Rightarrow\) \(\lambda\) is different
• Example 1: Cobb-Douglas utility

\[ \min p_1 x_1 + p_2 x_2 \]
\[ s.t. \ x_1^\alpha x_2^{1-\alpha} \geq \bar{u} \]

• Lagrangean =

• F.o.c.:

• Solution: \( h_1^* = \), \( h_2^* = \)

• \( \partial h_i^*/\partial p_i < 0, \partial h_i^*/\partial p_j > 0, j \neq i \)
5 Slutsky equation: Intuition

• Now: go back to Utility Max. in case where $p_2$ increases to $p'_2 > p_2$

• What is $\partial x_2^*/\partial p_2$? Decompose effect:

  1. Substitution effect of an increase in $p_i$
     - $\partial h_2^*/\partial p_2$, that is change in EMIN point as $p_2$ decreases
     - Moving along an indifference curve
     - Certainly $\partial h_2^*/\partial p_2 < 0$
2. Income effect of an increase in $p_i$

- $\partial x_2^*/\partial M$, increase in consumption of good 2 due to increased income

- Shift out a budget line

- $\partial x_2^*/\partial M > 0$ for normal goods, $\partial x_2^*/\partial M < 0$ for inferior goods
6 Next Lectures

• More comparative statics:
  – Intuition
  – Slutzky Equation

• Then moving on to applications:
  – Labor Supply
  – Intertemporal choice
  – Economics of Altruism