Economics 101A
(Lecture 2)

Stefano DellaVigna

September 1, 2005
Outline

1. Who Am I?

2. Questions on Syllabus

3. An Example: Economics of Discrimination

4. Comparative Statics

5. Implicit function theorem

6. Envelope Theorem
1 Who am I?

Stefano DellaVigna

• Assistant Professor, Department of Economics

• Bocconi (Italy) undergraduate (Econ.), Harvard PhD (Econ.)

• Psychology and economics, applied microeconomics, behavioral finance, aging, media

• Evans 515

• OH: We 2-4
Questions on Syllabus?

For questions on enrollment, note:

- In the past, everyone intending to take the class managed to

- I expect (and hope) that this will happen also this year

- However: No certainty of this

- Have to wait till end of second week

- For further questions, see Desiree Schaan. OH: 508-2, 10-12, 1-3 every day till September 7th
3 An Example: Economics of discrimination

- Ok, I need maths. But where is the economics?

- Workers:
  - A and B. They produce 1 widget per hour
  - Both have reservation wage $\bar{u}$

- Firm:
  - sells widgets at price $p > \bar{u}$ (assume $p$ given)
  - dislikes worker B
  - Maximizes profits ($p \times$ no of widgets – cost of labor) minus disutility $d$ if employs B
• Wages and employment in this industry?

• Employment
  
  – Net surplus from employing A: $p - \bar{u}$
  
  – Net surplus from employing B: $p - \bar{u} - d$
  
  – If $\bar{u} < p < \bar{u} + d$, Firm employs A but not B
  
  – If $\bar{u} + d < p$, Firm employs both

• What about wages?
• Case I. Firm monopolist and no worker union
  – Firm maximizes profits and gets all the net surplus
  – Wages of A and B equal $\bar{u}$

• Case II. Firm monopolist and worker union
  – Firm and worker get half of the net surplus each
  – Wage of A equals $\bar{u} + .5 \times (p - \bar{u})$
  – Wage of B equals $\bar{u} + .5 \times (p - \bar{u} - d)$

• Case III. Perfect competition among firms that discriminate ($d > 0$)
  – Prices are lowered to the cost of production
  – Wage of A equals $p$
  – $B$ is not employed
• The magic of competition

• Case IIIb. Perfect competition + At least one firm does not discriminate \((d = 0)\)
  
  - This firm offers wage \(p\) to both workers
  
  - What happens to worker \(B\)?
  
  - She goes to the firm with \(d = 0\)!
  
  - In equilibrium now:
    
    * Wage of \(A\) equals \(p\)

    * Wage of \(B\) equals \(p\) as well!
• Is this true? Any evidence?

• S. Black and P. Strahan, AER 2001.
  - Local monopolies in banking industry until mid 70s
  - Mid 70s: deregulation
  - From local monopolies to perfect competition.
  - Wages?
    * Wages fall by 6.1 percent
  - Discrimination?
    * Wages fall by 12.5 percent for men
    * Wages fall by 2.9 percent for women
    * Employment of women as managers increases by 10 percent
• More evidence on discrimination

• Does black-white and male-female wage back derive from discrimination?

• Field experiment (Betrand and Mullainathan, 2005)

• Send real CV with randomly picked names:
  – Male/Female
  – White/African American

• Measure call-back rate from interview

• Results (Table 1, Handout):
  – Call-back rates 50 percent higher for Whites!
  – No effect for Male-Female call back rates
• Strong evidence of discrimination against African Americans

• Example of Applied Microeconomics

• Not (really) covered in this class: See Ec142 and (partly) Ec152

• If curious: read Steven Levitt and Stephen Dubner, *Freakonomics*. 
4 Comparative statics

• Economics is all about ‘comparative statics’

• What happens to optimal economic choices if we change one parameter?

• Example: Car production. Consumer:
  1. Car purchase and increase in oil price
  2. Car purchase and increase in income

• Producer:
  1. Car production and minimum wage increase
  2. Car production and decrease in tariff on Japanese cars

• Next two sections
5 Implicit function theorem

- Implicit function: Ch. 2, pp. 32–33 [OLD, 32–34]

- Consider function \( y = g(x, p) \)

- Can rewrite as \( y - g(x, p) = 0 \)

- **Implicit function** has form: \( h(y, x, p) = 0 \)

- Often we need to go from implicit to explicit function

- Example 3: \( 1 - xy - e^y = 0 \).

- Write \( x \) as function of \( y \):

- Write \( y \) as function of \( x \):
• **Univariate implicit function theorem (Dini):** Consider an equation \( f(p, x) = 0 \), and a point \((p_0, x_0)\) solution of the equation. Assume:

1. \( f \) continuous and differentiable in a neighbourhood of \((p_0, x_0)\);
2. \( f_x'(p_0, x_0) \neq 0 \).

• Then:

1. There is one and only function \( x = g(p) \) defined in a neighbourhood of \( p_0 \) that satisfies \( f(p, g(p)) = 0 \) and \( g(p_0) = x_0 \);
2. The derivative of \( g(p) \) is

\[
g'(p) = -\frac{f_p'(p, g(p))}{f_x'(p, g(p))}
\]
• Example 3 (continued): \(1 - xy - e^y = 0\)

• Find derivative of \(y = g(x)\) implicitly defined for \((x, y) = (1, 0)\)

• Assumptions:
  1. Satisfied?
  2. Satisfied?

• Compute derivative
• **Multivariate implicit function theorem (Dini):** Consider a set of equations \( f_1(p_1, \ldots, p_n; x_1, \ldots, x_s) = 0; \ldots; f_s(p_1, \ldots, p_n; x_1, \ldots, x_s) = 0 \), and a point \((p_0, x_0)\) solution of the equation. Assume:

1. \( f_1, \ldots, f_s \) continuous and differentiable in a neighbourhood of \((p_0, x_0)\);

2. The following Jacobian matrix \( \frac{\partial f}{\partial x} \) evaluated at \((p_0, x_0)\) has determinant different from 0:

\[
\frac{\partial f}{\partial x} = \begin{pmatrix}
\frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_s} \\
\vdots & \vdots & \vdots \\
\frac{\partial f_s}{\partial x_1} & \frac{\partial f_s}{\partial x_s}
\end{pmatrix}
\]
Then:

1. There is one and only set of functions $x = g(p)$ defined in a neighbourhood of $p_0$ that satisfy $f(p, g(p)) = 0$ and $g(p_0) = x_0$;

2. The partial derivative of $x_i$ with respect to $p_k$ is

$$
\frac{\partial g_i}{\partial p_k} = -\frac{\det \left( \frac{\partial (f_1, \ldots, f_s)}{\partial (x_1, \ldots, x_{i-1}, p_k, x_{i+1}, \ldots, x_s)} \right)}{\det \left( \frac{\partial f}{\partial x} \right)}
$$
• Example 2 (continued): \( \text{Max } h(x_1, x_2) = p_1 * x_1^2 + p_2 * x_2^2 - 2x_1 - 5x_2 \)

• f.o.c. \( x_1 : 2p_1 * x_1 - 2 = 0 = f_1(p,x) \)

• f.o.c. \( x_2 : 2p_2 * x_2 - 5 = 0 = f_2(p,x) \)

• Comparative statics of \( x_1^* \) with respect to \( p_1 \)?

• First compute \( \text{det} \left( \frac{\partial f}{\partial x} \right) \)

\[
\left( \begin{array}{cc}
\frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\
\frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2}
\end{array} \right) = \left( \begin{array}{cc}
\text{ } & \\
\text{ } & 
\end{array} \right)
\]
• Then compute $\det \left( \frac{\partial (f_1, \ldots, f_s)}{\partial (x_1, \ldots, x_{i-1}, p_k, x_{i+1}, \ldots, x_s)} \right)$

\[
\begin{pmatrix}
\frac{\partial f_1}{\partial p_1} & \frac{\partial f_1}{\partial x_2} \\
\frac{\partial f_2}{\partial p_1} & \frac{\partial f_2}{\partial x_2}
\end{pmatrix} = \left( \begin{array}{c}
\vdots \\
\vdots
\end{array} \right)
\]

• Finally, $\frac{\partial x_1}{\partial p_1} =$

• Why did you compute $\det \left( \frac{\partial f}{\partial x} \right)$ already?
6 Envelope Theorem

• Ch. 2, pp. 33–37 [OLD, 34–39]

• You now know how $x_1^*$ varies if $p_1$ varies.

• How does $h(x^*(p))$ vary as $p_1$ varies?

• Differentiate $h(x_1^*(p_1, p_2), x_2^*(p_1, p_2), p_1, p_2)$ with respect to $p_1$:

$$\frac{dh(x_1^*(p_1, p_2), x_2^*(p_1, p_2), p_1, p_2)}{dp_1} = \frac{\partial h(x^*, p)}{\partial x_1} * \frac{\partial x_1^*(x^*, p)}{\partial p_1} + \frac{\partial h(x^*, p)}{\partial x_2} * \frac{\partial x_2^*(x^*, p)}{\partial p_1} + \frac{\partial h(x^*, p)}{\partial p_1}$$

• Notice: First two terms are zero.
• **Envelope Theorem** for unconstrained maximization. Assume that you maximize function $f(x; p)$ with respect to $x$. Consider then the function $f$ at the optimum, that is, $f(x^*(p), p)$. The total differential of this function with respect to $p_i$ equals the partial derivative with respect to $p_i$:

$$
\frac{df(x^*(p), p)}{dp_i} = \frac{\partial f(x^*(p), p)}{\partial p_i}.
$$

• You can disregard the indirect effects. Graphical intuition.
7 Next Class

• Next class:
  – Convexity and Concavity
  – Constrained Maximization
  – Envelope Theorem II

• Going toward:
  – Preferences
  – Utility Maximization (where we get to apply maximization techniques the first time)