Outline

1. Utility Maximization – Tricky Cases (cntd)

2. Comparative Statics (introduction)

3. Income changes

4. Price Changes

5. Expenditure minimization

6. Slutsky Equation: Intuition
1 Utility maximization – tricky cases

1. Solution does not satisfy $x_1^* > 0$ or $x_2^* > 0$. Example:

$$\text{max } x_1^* (x_2 + 5)$$
$$s.t. \ p_1 x_1 + p_2 x_2 = M$$

- In this case consider corner conditions: what happens for $x_1^* = 0$? And $x_2^* = 0$?
2 Comparative Statics (introduction)


- Utility maximization yields $x^*_i = x^*_i(p_1, p_2, M)$

- Quantity consumed as a function of income and price

- What happens to quantity consumed $x^*_i$ as prices or income varies?
• Simple case: Equal increase in prices and income.

• $M' = tM$, $p'_1 = tp_1$, $p'_2 = tp_2$.

• Compare $x^*(tM, tp_1, tp_2)$ and $x^*(M, p_1, p_2)$.

• What happens?

• Write budget line: $tp_1 x_1 + tp_2 x_2 = tM$

• Demand is homogeneous of degree 0 in $p$ and $M$:

$$x^*(tM, tp_1, tp_2) = t^0 x^*(M, p_1, p_2) = x^*(M, p_1, p_2).$$
• Consider Cobb-Douglas Case:

\[ x_1^* = \frac{\alpha}{\alpha + \beta} \frac{M}{p_1}, \quad x_2^* = \frac{\beta}{\alpha + \beta} \frac{M}{p_2} \]

• What is \( \partial x_1^*/\partial M \)?

• What is \( \partial x_1^*/\partial p_1 \)?

• What is \( \partial x_1^*/\partial p_2 \)?

• General results?
3 Income changes

- Income increases from $M$ to $M' > M$.

- Budget line $(p_1 x_1 + p_2 x_2 = M)$ shifts out:
  
  $$x_2 = \frac{M'}{p_2} - x_1 \frac{p_1}{p_2}$$

- New optimum?
• Engel curve: $x_i^*(M)$: demand for good $i$ as function of income $M$ holding fixed prices $p_1, p_2$

• Does $x_i^*$ increase with $M$?
  
  – Yes. Good $i$ is normal

  – No. Good $i$ is inferior
4 Price changes

• Price of good $i$ increases from $p_i$ to to $p_i' > p_i$

• For example, decrease in price of good 2, $p_2' < p_2$

• Budget line tilts:

$$x_2 = \frac{M}{p_2'} - x_1 \frac{p_1}{p_2'}$$

• New optimum?
• Demand curve: \( x_i^*(p_i) \): demand for good \( i \) as function of own price holding fixed \( p_j \) and \( M \)

• Odd convention of economists: plot price \( p_i \) on vertical axis and quantity \( x_i \) on horizontal axis. Better get used to it!
• Does $x_i^*$ decrease with $p_i$?
  
  – Yes. Most cases

  – No. Good $i$ is *Giffen*

  – Ex.: Potatoes in Ireland

  – Do not confuse with Veblen effect for luxury goods or informational asymmetries: these effects are real, but not included in current model
5 Expenditure minimization


- Solve problem EMIN (minimize expenditure):

\[
\min p_1 x_1 + p_2 x_2 \\
\text{s.t. } u(x_1, x_2) \geq \bar{u}
\]

- Choose bundle that attains utility $\bar{u}$ with minimal expenditure

- Ex.: You are choosing combination CDs/restaurant to make a friend happy

- If utility $u$ strictly increasing in $x_i$, can maximize s.t. equality

- Denote by $h_i(p_1, p_2, \bar{u})$ solution to EMIN problem

- $h_i(p_1, p_2, \bar{u})$ is Hicksian or compensated demand
• Graphically:
  – Fix indifference curve at level $\bar{u}$
  – Consider budget sets with different $M$
  – Pick budget set which is tangent to indifference curve

• Optimum coincides with optimum of Utility Maximization!

• Formally:
  \[
  h_i(p_1, p_2, \bar{u}) = x_i^*(p_1, p_2, e(p_1, p_2, \bar{u}))
  \]
Expenditure function is expenditure at optimum

\[ e(p_1, p_2, \bar{u}) = p_1 h_1(p_1, p_2, \bar{u}) + p_2 h_2(p_1, p_2, \bar{u}) \]

\( h_i(p_i) \) is *Hicksian or compensated demand* function

Is \( h_i \) always decreasing in \( p_i \)? Yes!

Graphical proof: moving along a convex indifference curve

(For non-convex indifferent curves, still true)
• Using first order conditions:

\[ L(x_1, x_2, \lambda) = p_1 x_1 + p_2 x_2 - \lambda (u(x_1, x_2) - \bar{u}) \]

\[ \frac{\partial L}{\partial x_i} = p_i - \lambda u'_i(x_1, x_2) = 0 \]

• Write as ratios:

\[ \frac{u'_1(x_1, x_2)}{u'_2(x_1, x_2)} = \frac{p_1}{p_2} \]

• \( MRS = \) ratio of prices as in utility maximization!

• However: different constraint \( \Rightarrow \) \( \lambda \) is different
Example 1: Cobb-Douglas utility

\[
\min p_1 x_1 + p_2 x_2 \\
\text{s.t. } x_1^\alpha x_2^{1-\alpha} \geq u
\]

Lagrangean =

F.o.c.:

Solution: \( h_1^* = \), \( h_2^* = \)

\( \frac{\partial h_i^*}{\partial p_i} < 0, \frac{\partial h_i^*}{\partial p_j} > 0, j \neq i \)
6 Slutsky equation: Intuition

- Now: go back to Utility Max. in case where \( p_2 \) increases to \( p'_2 > p_2 \)

- What is \( \partial x^*_2 / \partial p_2 \)? Decompose effect:

  1. Substitution effect of an increase in \( p_i \)

    - \( \partial h^*_2 / \partial p_2 \), that is change in EMIN point as \( p_2 \) decreases

    - Moving along an indifference curve

    - Certainly \( \partial h^*_2 / \partial p_2 < 0 \)
2. Income effect of an increase in $p_i$

- $\frac{\partial x^*_2}{\partial M}$, increase in consumption of good 2 due to increased income

- Shift out a budget line

- $\frac{\partial x^*_2}{\partial M} > 0$ for normal goods, $\frac{\partial x^*_2}{\partial M} < 0$ for inferior goods
7 Next Lectures

• More comparative statics:
  – Intuition
  – Slutzky Equation

• Then moving on to applications:
  – Labor Supply
  – Intertemporal choice
  – Economics of Altruism