Outline

1. Slutzky equation

2. Complements and substitutes

3. Do utility functions exist?

4. Application 1: Labor Supply
1 Slutsky equation

• Now: go back to Utility Max. in case where $p_2$ increases to $p'_2 > p_2$

• What is $\partial x^*_2/\partial p_2$? Decompose effect:

1. Substitution effect of an increase in $p_i$
   - $\partial h^*_2/\partial p_2$, that is change in EMIN point as $p_2$ descreases
   - Moving along an indifference curve
   - Certainly $\partial h^*_2/\partial p_2 < 0$
2. Income effect of an increase in $p_i$

- $\partial x_2^*/\partial M$, increase in consumption of good 2 due to increased income

- Shift out a budget line

- $\partial x_2^*/\partial M > 0$ for normal goods, $\partial x_2^*/\partial M < 0$ for inferior goods
• Nicholson, Ch. 5, pp. 135–138 [OLD: 131–136].

• \( h_i(p_1, p_2, \bar{u}) = x_i^*(p_1, p_2, e(p_1, p_2, \bar{u})) \)

• How does the Hicksian demand change if price \( p_i \) changes?

\[
\frac{dh_i}{dp_i} = \frac{\partial x_i^*(p, e)}{\partial p_i} + \frac{\partial x_i^*(p, e)}{\partial M} \frac{\partial e(p, \bar{u})}{\partial p_i}
\]

• What is \( \frac{\partial e(p, \bar{u})}{\partial p_i} \)? Envelope theorem:

\[
\frac{\partial e(p, \bar{u})}{\partial p_i} = \frac{\partial}{\partial p_i} [p_1h_1^* + p_2h_2^* - \lambda(u(h_1^*, h_2^*, \bar{u}) - \bar{u})] = h_i^*(p_1, p_2, \bar{u}) = x_i^*(p_1, p_2, e(p, \bar{u}))
\]
Therefore

\[
\frac{\partial h_i(p, \bar{u})}{\partial p_i} = \frac{\partial x_i^*(p, e)}{\partial p_i} + \frac{\partial x_i^*(p, e)}{\partial M} x_1^*(p_1, p_2, e)
\]

Rewrite as

\[
\frac{\partial x_i^*(p, M)}{\partial p_i} = \frac{\partial h_i(p, v(p, M))}{\partial p_i}
\]

\[
- x_1^*(p_1, p_2, M) \frac{\partial x_i^*(p, M)}{\partial M}
\]

Important result! Allows decomposition into substitution and income effect
• Two effects of change in price:

1. Substitution effect negative: \( \frac{\partial h_i(p, v(p, M))}{\partial p_i} < 0 \)

2. Income effect: \( -x_i^*(p_1, p_2, M) \frac{\partial x_i^*(p, M)}{\partial M} \)
   - negative if good \( i \) is normal (\( \frac{\partial x_i^*(p, M)}{\partial M} > 0 \))
   - positive if good \( i \) is inferior (\( \frac{\partial x_i^*(p, M)}{\partial M} < 0 \))

• Overall, sign of \( \frac{\partial x_i^*(p, M)}{\partial p_i} \)?
   - negative if good \( i \) is normal
   - it depends if good \( i \) is inferior
• Example 1 (ctd.): Cobb-Douglas. Apply Slutsky equation

• \( x_i^* = \alpha M/p_i \)

• \( h_i^* = \)

• Derivative of Hicksian demand with respect to price:

\[
\frac{\partial h_i(p, u)}{\partial p_i} = 
\]

• Rewrite \( h_i^* \) as function of \( m \): \( h_i(p, v(p, M)) \)

• Compute \( v(p, M) = \)
• Substitution effect:

\[ \frac{\partial h_i(p, v(p, M))}{\partial p_i} = \]

• Income effect:

\[ -x_i^*(p_1, p_2, M) \frac{\partial x_i^*(p, M)}{\partial M} = \]

• Sum them up to get

\[ \frac{\partial x_i^*(p, M)}{\partial p_i} = \]

• It works!
2 Complements and substitutes

• Nicholson, Ch. 6, pp. 161–166 [OLD: 152–158].

• How about if price of another good changes?

• Generalize Slutsky equation

• Slutsky Equation:

\[
\frac{\partial x_i^* (p, M)}{\partial p_j} = \frac{\partial h_i (p, v(p, M))}{\partial p_j} - x_j^* (p_1, p_2, M) \frac{\partial x_i^* (p, M)}{\partial M}
\]
• Substitution effect

\[ \frac{\partial h_i(p, v(p, M))}{\partial p_j} > 0 \]

for \( n = 2 \) (two goods). Ambiguous for \( n > 2 \).

• Income effect:

\[ -x_j^*(p_1, p_2, M) \frac{\partial x_i^*(p, M)}{\partial M} \]

– negative if good \( i \) is normal

– positive if good \( i \) is inferior

• How do we define complements and substitutes?
• Def. 1. Goods $i$ and $j$ are **gross substitutes** at price $p$ and income $M$ if
\[
\frac{\partial x_i^*(p, M)}{\partial p_j} > 0
\]

• Def. 2. Goods $i$ and $j$ are **gross complements** at price $p$ and income $M$ if
\[
\frac{\partial x_i^*(p, M)}{\partial p_j} < 0
\]

• Example 1 (ctd.): $x_1^* = \alpha M/p_1$, $x_2^* = \beta M/p_2$.

• Gross complements or gross substitutes? Neither!

• Notice: $\frac{\partial x_i^*(p, M)}{\partial p_j}$ is usually different from $\frac{\partial x_j^*(p, M)}{\partial p_i}$
• Better definition.

• Def. 3. Goods $i$ and $j$ are **net substitutes** at price $p$ and income $M$ if

$$\frac{\partial h_i^* (p, v(p, M))}{\partial p_j} = \frac{\partial h_j^* (p, v(p, M))}{\partial p_i} > 0$$

• Def. 4. Goods $i$ and $j$ are **net complements** at price $p$ and income $M$ if

$$\frac{\partial h_i^* (p, v(p, M))}{\partial p_j} = \frac{\partial h_j^* (p, v(p, M))}{\partial p_i} < 0$$

• Example 1 (ctd.): $h_1^* = \bar{u} \left( \frac{\alpha p_2}{1-\alpha p_1} \right)^{1-\alpha}$

• Net complements or net substitutes? Net substitutes!
3 Do utility functions exist?

- Preferences and utilities are theoretical objects
- Many different ways to write them
- How do we tie them to the world?
- Use actual choices – revealed preferences approach
• Typical economists’ approach. Compromise of:
  
  – realism
  
  – simplicity

• Assume a class of utility functions (CES, Cobb-Douglas...) with free parameters

• Estimate the parameters using the data
4 Labor Supply I

- Nicholson Ch. 16, pp. 477–484 [OLD: Ch. 22, pp. 606–613.]

- Labor supply decision: how much to work in a day.

- Goods: consumption good $c$, hours worked $h$

- Price of good $p$, hourly wage $w$

- Consumer spends $24 - h = l$ hours in units of leisure

- Utilify function: $u(c, l)$
• Budget constraint?

• Income of consumer: \( M + wh = M + w(24 - l) \)

• Budget constraint: \( pc \leq M + w(24 - l) \) or
  \[ pc + wl \leq M + 24w \]

• Notice: leisure \( l \) is a consumption good with price \( w \). Why?

• General category: opportunity cost

• Instead of enjoying one hour of TV, I could have worked one hour and gained wage \( w \).

• You should value the marginal hour of TV \( w \)!
• Opportunity costs are very important!

• Example 2. CostCo has a warehouse in SoMa

• SoMa used to have low cost land, adequate for warehouses

• Price of land in SoMa triples in 10 years.

• Should firm relocate the warehouse?
• Did costs of staying in SoMa go up?

• No.

• Did the opportunity cost of staying in SoMa go up?

• Yes!

• Firm can sell at high price and purchase land in cheaper area.
• Let’s go back to labor supply

• Maximization problem is

\[
\max u(c, l) \\
\text{s.t. } pc + wl \leq M + 24w
\]

• Standard problem (except for 24w)

• First order conditions

• Assume utility function Cobb-Douglas:

\[
u(c, l) = c^{\alpha} l^{1-\alpha}
\]
• Solution is

\[ c^* = \alpha \frac{M + 24w}{p} \]

\[ l^* = (1 - \alpha) \left( 24 + \frac{M}{w} \right) \]

• Both \( c \) and \( l \) are normal goods

• Unlike in standard Cobb-Douglas problems, \( c^* \) depends on price of other good \( w \)

• Why? Agents are endowed with \( M \) AND 24 hours of \( l \) in this economy

• Normally, agents are only endowed with \( M \)
5 Next Lectures

• More applications:
  – Intertemporal choice
  – Economics of Altruism