Outline

1. Mid-Term Suggestions

2. Nobel Prize winners

3. Risk Aversion and Lottery

4. Insurance

5. Investment in Risky Asset

6. Measures of Risk Aversion
1 Mid-Term Suggestions

• Suggestions from you...
2 Nobel Prize winners

• Robert Aumann (Hebrew University)

• Thomas Schelling (University of Maryland)

• Game Theory

• (Coming in third part of course...)

• Repeated Games and Applications to Conflict
3 Risk Aversion and Lottery

- Are you risk-averse?

- Let’s see...
4 Insurance

- Nicholson, Ch. 18, pp. 545–551 [OLD: Ch. 8, pp. 211-216] Notice: different treatment than in class

- Individual has:
  
  - wealth $w$
  
  - utility function $u$, with $u' > 0$, $u'' < 0$

- Probability $p$ of accident with loss $L$

- Insurance offers coverage:
  
  - premium $q$ for each $1$ paid in case of accident
  
  - units of coverage purchased $\alpha$
• Individual maximization:

$$\max_{\alpha} (1 - p) u(w - q\alpha) + pu(w - q\alpha - L + \alpha)$$

s.t. $\alpha \geq 0$

• Assume $\alpha^* \geq 0$, check later

• First order conditions:

$$0 = -q(1 - p) u'(w - q\alpha) + (1 - q) pu'(w - q\alpha - L + \alpha)$$

or

$$\frac{u'(w - q\alpha)}{u'(w - q\alpha - L + \alpha)} = \frac{1 - q}{q} \frac{p}{1 - p}.$$ 

• Assume first $q = p$ (insurance is fair)

• Solution for $\alpha^* = ?$
• $\alpha^* > 0$, so we are ok!

• What if $q > p$ (insurance needs to cover operating costs)?

• Insurance will be only partial (if at all)

• Exercise: Check second order conditions!
5 Investment in Risk Asset

- Individual has:
  - wealth $w$
  - utility function $u$, with $u' > 0$

- Two possible investments:
  - Asset B (bond) yields return 1 for each dollar
  - Asset S (stock) yields uncertain return $(1 + r)$:
    * $r = r_+ > 0$ with probability $p$
    * $r = r_- < 0$ with probability $1 - p$
    * $Er = pr_+ + (1 - p)r_- > 0$

- Share of wealth invested in stock $S = \alpha$
• Individual maximization:

\[
\max_{\alpha} (1 - p) u (w [(1 - \alpha) + \alpha (1 + r_-)]) + \\
+ pu (w [(1 - \alpha) + \alpha (1 + r_+)]) \\
s.t. 0 \leq \alpha \leq 1
\]

• Case of risk neutrality: \( u(x) = a + bx, \ b > 0 \)

• Assume \( a = 0 \) (no loss of generality)

• Maximization becomes

\[
\max_{\alpha} b (1 - p) (w [1 + \alpha r_-]) + bp (w [1 + \alpha r_+]) \\
or \\
\max_{\alpha} bw + \alpha bw [(1 - p) r_- + pr_+]
\]

• Sign of term in square brackets? Positive!

• Set \( \alpha^* = 1 \)
• Case of risk aversion: $u'' < 0$

• Assume $0 \leq \alpha^* \leq 1$, check later

• First order conditions:

$$0 = (1 - p) (\omega r_-) u' (w [1 + \alpha r_-]) + p (\omega r_+) u' (w [1 + \alpha r_+])$$

• Can $\alpha^* = 0$ be solution?

• Solution is $\alpha^* > 0$ (positive investment in stock)

• Exercise: Check s.o.c.
6 Measures of Risk Aversion

- Nicholson, Ch. 18, pp. 541–545 [OLD: Ch. 8, pp. 207–210].

- How risk averse is an individual?

- Two measures:
  - Absolute Risk Aversion \( r_A \):
    \[
    r_A = -\frac{u''(x)}{u'(x)}
    \]
  - Relative Risk Aversion \( r_R \):
    \[
    r_R = -\frac{u''(x)}{u'(x)} x
    \]

- Examples in the Problem Set
7 Next lecture and beyond

• Tu:
  – Time consistency
  – Time Inconsistency
  – Application to health clubs

• Then:
  – Begin Production
  – Returns to scale
  – Cost minimization