Outline

1. Investment in Risky Asset II
2. Measures of Risk Aversion
3. Time Consistency
4. Time Inconsistency
1 Investment in Risk Asset II

• Individual has:
  
  – wealth \( w \)

  – utility function \( u \), with \( u' > 0 \)

• Two possible investments:
  
  – Asset B (bond) yields return 1 for each dollar
  
  – Asset S (stock) yields uncertain return \((1 + r)\):
  
    \* \( r = r_+ > 0 \) with probability \( p \)
    
    \* \( r = r_- < 0 \) with probability \( 1 - p \)

    \* \( Er = pr_+ + (1 - p)r_- > 0 \)

• Share of wealth invested in stock \( S = \alpha \)
• Individual maximization:

\[
\max_{\alpha} (1 - p) u \left( w \left[ (1 - \alpha) + \alpha (1 + r_-) \right] \right) + \\
+ pu \left( w \left[ (1 - \alpha) + \alpha (1 + r_+) \right] \right) \\
\text{s.t. } 0 \leq \alpha \leq 1
\]

• Case of risk neutrality: \( u(x) = a + bx, \ b > 0 \)

• Assume \( a = 0 \) (no loss of generality)

• Maximization becomes

\[
\max_{\alpha} b (1 - p) \left( w [1 + \alpha r_-] \right) + bp \left( w [1 + \alpha r_+] \right)
\]

or

\[
\max_{\alpha} bw + \alpha bw \left[ (1 - p) r_- + pr_+ \right]
\]

• Sign of term in square brackets? Positive!

• Set \( \alpha^* = 1 \)
• Case of risk aversion: \( u'' < 0 \)

• Assume \( 0 \leq \alpha^* \leq 1 \), check later

• First order conditions:

\[
0 = (1 - p) (wr_-) u' (w [1 + \alpha r_-]) + \\
+ p (wr_+) u' (w [1 + \alpha r_+])
\]

• Can \( \alpha^* = 0 \) be solution?

• Solution is \( \alpha^* > 0 \) (positive investment in stock)

• Exercise: Check s.o.c.
2 Measures of Risk Aversion

• Nicholson, Ch. 18, pp. 541–545 [OLD: Ch. 8, pp. 207–210].

• How risk averse is an individual?

• Two measures:

  – Absolute Risk Aversion \( r_A \):

    \[
    r_A = - \frac{u''(x)}{u'(x)}
    \]

  – Relative Risk Aversion \( r_R \):

    \[
    r_R = - \frac{u''(x)}{u'(x)} x
    \]

• Examples in the Problem Set
3 Time consistency

• Intertemporal choice

• Three periods, $t = 0$, $t = 1$, and $t = 2$

• At each period $i$, agents:
  
  – have income $M'_i = M_i + \text{savings/debts from previous period}$
  
  – choose consumption $c_i$
  
  – can save/borrow $M'_i - c_i$
  
  – no borrowing in last period: at $t = 2$ $M'_2 = c_2$
• Utility function at $t = 0$

$$u(c_0, c_1, c_2) = U(c_0) + \frac{1}{1 + \delta}EU(c_1) + \frac{1}{(1 + \delta)^2}EU(c_2)$$

• Utility function at $t = 1$

$$u(c_1, c_2) = U(c_1) + \frac{1}{1 + \delta}EU(c_2)$$

• Utility function at $t = 2$

$$u(c_2) = U(c_2)$$

• $U' > 0$, $U'' < 0$
• Question: Do preferences of agent in period 0 agree with preferences of agent in period 1?

• Period 1.

• Budget constraint at $t = 1$:

$$c_1 + \frac{1}{1 + r}c_2 \leq M_1' + \frac{1}{1 + r}M_2$$

• Maximization problem:

$$\max U(c_1) + \frac{1}{1 + \delta}EU(c_2)$$

$$\text{s.t. } c_1 + \frac{1}{1 + r}c_2 \leq M_1' + \frac{1}{1 + r}M_2$$

• First order conditions:

• Ratio of f.o.c.s:

$$\frac{U''(c_1)}{EU''(c_2)} = \frac{1 + r}{1 + \delta}$$
• Back to period 0.

• Agent at time 0 can commit to consumption at time 1 as function of uncertain income $M_1$.

• Anticipated budget constraint at $t = 1$:

$$c_1 + \frac{1}{1 + r} c_2 \leq M_1' + \frac{1}{1 + r} M_2$$

• Maximization problem:

$$\max U(c_0) + \frac{1}{1 + \delta} U(c_1) + \frac{1}{(1 + \delta)^2} EU(c_2)$$

$$s.t. \ c_1 + \frac{1}{1 + r} c_2 \leq M_1' + \frac{1}{1 + r} M_2$$

• First order conditions:

• Ratio of f.o.c.s:

$$\frac{U''(c_1)}{EU'(c_2)} = \frac{1 + r}{1 + \delta}$$
• The two conditions coincide!

• **Time consistency.** Plans for future coincide with future actions.

• To see why, rewrite utility function $u(c_0, c_1, c_2)$:

$$
U(c_0) + \frac{1}{1+\delta} U(c_1) + \frac{1}{(1+\delta)^2} EU(c_2)
$$

$$
= U(c_0) + \frac{1}{1+\delta} \left[ U(c_1) + \frac{1}{1+\delta}EU(c_2) \right]
$$

• Expression in brackets coincides with utility at $t = 1$

• Is time consistency right?
  
  – addictive products (alcohol, drugs);

  – good actions (exercising, helping friends);

  – immediate gratification (shopping, credit card borrowing)
4 Time Inconsistency

- Alternative specification (Akerlof, 1991; Laibson, 1997; O’Donoghue and Rabin, 1999)

- Utility at time $t$ is $u(c_t, c_{t+1}, c_{t+2})$:

$$u(c_t) + \frac{\beta}{1 + \delta} u(c_{t+1}) + \frac{\beta}{(1 + \delta)^2} u(c_{t+2}) + ...$$

- Discount factor is

$$1, \frac{\beta}{1 + \delta'}, \frac{\beta}{(1 + \delta)^2'}, \frac{\beta}{(1 + \delta)^3'}, ...$$

instead of

$$1, \frac{1}{1 + \delta'}, \frac{1}{(1 + \delta)^2'}, \frac{1}{(1 + \delta)^3'}, ...$$

- What is the difference?

- Immediate gratification: $\beta < 1$
• Back to our problem: **Period 1.**

• Maximization problem:

\[
\begin{align*}
\max & \quad U(c_1) + \frac{\beta}{1 + \delta} EU(c_2) \\
\text{s.t.} & \quad c_1 + \frac{1}{1 + r} c_2 \leq M'_1 + \frac{1}{1 + r} M_2 
\end{align*}
\]

• First order conditions:

• Ratio of f.o.c.s:

\[
\frac{U'(c^*_1)}{EU'(c^*_2)} = \beta \frac{1 + r}{1 + \delta}
\]
• Now, **period 0** with commitment.

• Maximization problem:

\[
\begin{align*}
\max & \quad U(c_0) + \frac{\beta U(c_1)}{1 + \delta} + \frac{\beta}{(1 + \delta)^2} EU(c_2) \\
\text{s.t.} & \quad c_1 + \frac{1}{1 + r} c_2 \leq M_1' + \frac{1}{1 + r} M_2
\end{align*}
\]

• First order conditions:

• Ratio of f.o.c.s:

\[
\frac{U'(c_1^*,c)}{EU'(c_2^*,c)} = \frac{1 + r}{1 + \delta}
\]

• The two conditions differ!

• Time inconsistency: \( c_1^*,c < c_1^* \) and \( c_2^*,c > c_2^* \)

• The agent allows him/herself too much immediate consumption and saves too little
• Ok, we agree. but should we study this as economists?

• YES!
  – One trillion dollars in credit card debt;
  – Most debt is in teaser rates;
  – Two thirds of Americans are overweight or obese;
  – $10bn health-club industry

• Is this testable?
  – In the laboratory?
  – In the field?
5 Next Lecture

• An Example: Health club Attendance

• Cost Minimization

• Solve an Example

• Cases in which s.o.c. are not satisfied

• Start Profit Maximization