Economics 101A
(Lecture 15)

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Outline

1. Time Inconsistency II

2. Health Club Attendance

3. Production: Introduction

4. Production Function

5. Returns to Scale

6. Two-step Cost Minimization
1 Time Inconsistency II

• Alternative specification (Akerlof, 1991; Laibson, 1997; O’Donoghue and Rabin, 1999)

• Utility at time $t$ is $u(c_t, c_{t+1}, c_{t+2})$:

$$u(c_t) + \frac{\beta}{1 + \delta} u(c_{t+1}) + \frac{\beta}{(1 + \delta)^2} u(c_{t+2}) + \ldots$$

• Discount factor is

$$1, \frac{\beta}{1 + \delta'}, \frac{\beta}{(1 + \delta)^2}, \frac{\beta}{(1 + \delta)^3}, \ldots$$

instead of

$$1, \frac{1}{1 + \delta'}, \frac{1}{(1 + \delta)^2}, \frac{1}{(1 + \delta)^3}, \ldots$$

• What is the difference?

• *Immediate gratification*: $\beta < 1$
• Back to our problem: **Period 1**.

• Maximization problem:

\[
\max U(c_1) + \frac{\beta}{1 + \delta} EU(c_2)
\]

\[
s.t. \ c_1 + \frac{1}{1 + r} c_2 \leq M'_1 + \frac{1}{1 + r} M_2
\]

• First order conditions:

• Ratio of f.o.c.s:

\[
\frac{U'(c_1^*)}{EU'(c_2^*)} = \beta \frac{1 + r}{1 + \delta}
\]
• Now, period 0 with commitment.

• Maximization problem:

\[
\max U(c_0) + \frac{\beta}{1+\delta}U(c_1) + \frac{\beta}{(1+\delta)^2}EU(c_2)
\]

\[
s.t. c_1 + \frac{1}{1+r}c_2 \leq M_1' + \frac{1}{1+r}M_2
\]

• First order conditions:

• Ratio of f.o.c.s:

\[
\frac{U'(c_1^*,c)}{EU'(c_2^*,c)} = \frac{1+r}{1+\delta}
\]

• The two conditions differ!

• Time inconsistency: \(c_1^{*,c} < c_1^*\) and \(c_2^{*,c} > c_2^*\)

• The agent allows him/herself too much immediate consumption and saves too little
• Ok, we agree. but should we study this as economists?

• YES!
  – One trillion dollars in credit card debt;
  – Most debt is in teaser rates;
  – Two thirds of Americans are overweight or obese;
  – $10bn health-club industry

• Is this testable?
  – In the laboratory?
  – In the field?
2 Health Club Attendance

• Health club industry study (DellaVigna and Malmendier, 2002)

• 3 health clubs

• Data on attendance from swiping cards

• Choice of contracts:
  – Monthly contract with average price of $75
  – 10-visit pass for $100

• Consider users that choose monthly contract. Attendance?
• Attend on average 4.8 times per *month*

• Pay on average over $17

• Average delay of 2.2 months ($185) between last attendance and contract termination

• Over membership, user could have saved $700 by paying per visit
• Health club attendance:
  
  – immediate cost \( c \)
  
  – delayed benefit \( b \)

• At sign-up (attend tomorrow):

\[
NB^t = -\frac{\beta}{1 + \delta} c + \frac{\beta}{(1 + \delta)^2} b
\]

• Plan to attend if \( NB^t > 0 \)

\[
c < \frac{1}{(1 + \delta)} b
\]
• Once moment to attend comes:

\[ NB = -c + \frac{\beta}{(1 + \delta)}b \]

• Attend if \( NB > 0 \)

\[ c < \frac{\beta}{(1 + \delta)}b \]
• Interpretations?

• Users are buying a commitment device

• User underestimate their future self-control problems:
  – They overestimate future attendance
  – They delay cancellation
3 Production: Introduction

- Second half of the economy. Production

- Example. Ford and the Minivan (Petrin, 2002):
  - Ford had idea: "Mini/Max" (early '70s)
  - Did Ford produce it?
  - No!
  - Ford was worried of cannibalizing station wagon sector
  - Chrysler introduces Dodge Caravan (1984)
  - Chrysler: $1.5bn profits (by 1987)!
• Why need separate treatment?

• Perhaps firms maximize utility...

• ...we can be more precise:
  – Competition
  – Institutional structure
4 Production Function

• Nicholson, Ch. 7, pp. 183–190; 195–200 [OLD: Ch. 11, pp. 268–275; 280–285]

• Production function: $y = f(z)$. Function $f : \mathbb{R}_+^n \rightarrow \mathbb{R}_+$

• Inputs $z = (z_1, z_2, \ldots, z_n)$: labor, capital, land, human capital

• Output $y$: Minivan, Intel Pentium III, mangoes (Philippines)

• Properties of $f$:
  
  – no free lunches: $f(0) = 0$
  
  – positive marginal productivity: $f_i'(z) > 0$
  
  – decreasing marginal productivity: $f_{i,i}''(z) < 0$
• Isoquants \( Q (y) = \{ x | f (x) = y \} \)

• Set of inputs \( z \) required to produce quantity \( y \)

• Special case. Two inputs:
  
  - \( z_1 = L \) (labor)
  
  - \( z_2 = K \) (capital)

• Isoquant: \( f(L, K) - y = 0 \)

• Slope of isoquant \( dK/dL = MRTS \)
• Convex production function if convex isoquants

• Reasonable: combine two technologies and do better!

• Mathematically, \( \frac{d^2K}{d^2L} = \)
5  Returns to Scale

• Nicholson, Ch. 7, pp. 190–193 [OLD: Ch. 11, pp. 275–278]

• Effect of increase in labor: $f'_L$

• Increase of all inputs: $f(tz)$ with $t$ scalar, $t > 1$

• How much does input increase?
  
  – Decreasing returns to scale: for all $z$ and $t > 1$,

  \[ f(tz) < tf(z) \]

  – Constant returns to scale: for all $z$ and $t > 1$,

  \[ f(tz) = tf(z) \]
- Increasing returns to scale: for all $z$ and $t > 1$,

$$f(tz) > t f(z)$$
• Example: \( y = f(K, L) = AK^\alpha L^\beta \)

• Marginal product of labor: \( f'_L = \)

• Decreasing marginal product of labor: \( f''_L = \)

• \( MRTS = \)

• Convex isoquant?

• Returns to scale: \( f(tK, tL) = A(tK)^\alpha (tL)^\beta = t^{\alpha + \beta} AK^\alpha L^\beta = t^{\alpha + \beta} f(K, L) \)
6 Two-step Cost minimization

- Nicholson, pp. 212–220 [OLD, Ch. 12 , pp. 298–307] 

- Objective of firm: Produce output that generates maximal profit.

- Decompose problem in two:
  - Given production level $y$, choose cost-minimizing combinations of inputs
  - Choose optimal level of $y$.

- First step. Cost-Minimizing choice of inputs
• Two-input case: Labor, Capital

• Input prices:
  
  – Wage \( w \) is price of \( L \)
  
  – Interest rate \( r \) is rental price of capital \( K \)

• Expenditure on inputs: \( wL + rK \)

• Firm objective function:

\[
\min wL + rK \\
\text{s.t. } f(L, K) \geq y
\]
Compare with expenditure minimization for consumers

First order conditions:

\[ w - \lambda f'_L = 0 \]

and

\[ r - \lambda f'_K = 0 \]

Rewrite as

\[ \frac{f'_L (L^*, K^*)}{f'_K (L^*, K^*)} = \frac{w}{r} \]

MRTS (slope of isoquant) equals ratio of input prices
• Graphical interpretation
• Derived demand for inputs:

\[- L = L^*(w, r, y) \]

\[- K = K^*(w, r, y) \]

• Value function at optimum is **cost function:**

\[ c(w, r, y) = wL^*(r, w, y) + rK^*(r, w, y) \]
• *Second step.* Given cost function, choose optimal quantity of $y$ as well

• Price of output is $p$.

• Firm’s objective:

$$\max py - c(w, r, y)$$

• First order condition:

$$p - c'_y (w, r, y) = 0$$

• Price equals marginal cost – very important!
7 Next Lecture

- Continue Cost Minimization

- Solve an Example

- Cases in which s.o.c. are not satisfied

- Start Profit Maximization