Outline

1. Cost Minimization: Example II

2. Cost Curves and Supply Function

3. One-step Profit Maximization

4. Introduction to Market Equilibrium
1 Cost Minimization: Example II

• Continue example above: \( y = f(L, K) = AK^\alpha L^\beta \)

• Define \( B := w \left( \frac{w \alpha}{\frac{\alpha}{1}} \right)^{\frac{\alpha}{\alpha+\beta}} + r \left( \frac{w \alpha}{\frac{\alpha}{1}} \right)^{\frac{\beta}{\alpha+\beta}} \)

• Cost-minimizing output choice:

\[
\max py - B \left( \frac{y}{A} \right)^{\frac{1}{\alpha+\beta}}
\]

• First order condition:

\[
p - \frac{1}{\alpha + \beta} \frac{B}{A} \left( \frac{y}{A} \right)^{\frac{1-(\alpha+\beta)}{\alpha+\beta}} = 0
\]

• Second order condition:

\[
- \frac{1}{\alpha + \beta} \frac{1 - (\alpha + \beta)}{\alpha + \beta} \frac{B}{A^2} \left( \frac{y}{A} \right)^{\frac{1-2(\alpha+\beta)}{\alpha+\beta}} < 0
\]
Solution:

- $\alpha + \beta = 1$ (CRS):
  
  * S.o.c. equal to 0

  * Solution depends on $p$

  * For $p > \frac{1}{\alpha + \beta} \frac{B}{A}$, produce $y^* \to \infty$

  * For $p = \frac{1}{\alpha + \beta} \frac{B}{A}$, produce any $y^* \in [0, \infty)$

  * For $p < \frac{1}{\alpha + \beta} \frac{B}{A}$, produce $y^* = 0$
− $\alpha + \beta > 1$ (IRS):

* S.o.c. positive

* Solution of f.o.c. is a minimum!

* Solution is $y^* \to \infty$.

* Keep increasing production since higher production is associated with higher returns
- \( \alpha + \beta < 1 \) (DRS):

* s.o.c. negative. OK!

* Solution of f.o.c. is an interior optimum

* This is the only "well-behaved" case under perfect competition

* Here can define a supply function
2 Cost Curves

• Nicholson, Ch. 8, pp. 220–228; Ch. 9, pp. 256–259 [OLD: Ch. 12, pp. 307–312 and Ch. 13, pp. 342–346.]

• Marginal costs $MC = \partial c / \partial y \rightarrow$ Cost minimization
  
  $p = MC = \partial c (w, r, y) / \partial y$

• Average costs $AC = c / y \rightarrow$ Does firm break even?
  
  $\pi = py - c (w, r, y) > 0$ iff

  $\pi / y = p - c (w, r, y) / y > 0$ iff

  $c (w, r, y) / y = AC < p$

• Supply function. Portion of marginal cost $MC$
  above average costs. (price equals marginal cost)
• Assume only 1 input (expenditure minimization is trivial)

• Case 1. Production function. $y = L^\alpha$
  
  – Cost function? (cost of input is $w$):
    $$c(w, y) = wL^*(w, y) = wy^{1/\alpha}$$

  – Marginal cost?
    $$\frac{\partial c(w, y)}{\partial y} = \frac{1}{\alpha}wy^{(1-\alpha)/\alpha}$$

  – Average cost $c(w, y)/y$?
    $$\frac{c(w, y)}{y} = \frac{wy^{1/\alpha}}{y} = wy^{(1-\alpha)/\alpha}$$
• **Case 1a.** $\alpha > 1$. Plot production function, total cost, average and marginal. Supply function?

• **Case 1b.** $\alpha = 1$. Plot production function, total cost, average and marginal. Supply function?

• **Case 1c.** $\alpha < 1$. Plot production function, total cost, average and marginal. Supply function?
• **Case 2.** *Non-convex technology.* Plot production function, total cost, average and marginal. Supply function?

• **Case 3.** *Technology with setup cost.* Plot production function, total cost, average and marginal. Supply function?
2.1 Supply Function

- Supply function: \( y^* = y^* (w, r, p) \)

- What happens to \( y^* \) as \( p \) increases?

- Is the supply function upward sloping?

- Remember f.o.c:
  \[
  p - c_y' (w, r, y) = 0
  \]

- Implicit function:
  \[
  \frac{\partial y^*}{\partial p} = - \frac{1}{-c_{y,y}'' (w, r, y)} > 0
  \]
  as long as s.o.c. is satisfied.

- Yes! Supply function is upward sloping.
3 One-step Profit Maximization

• Nicholson, Ch. 9, pp. 265–270 [OLD: Ch. 13, pp. 346–350].

• One-step procedure: maximize profits

• Perfect competition. Price $p$ is given
  
  – Firms are small relative to market
  
  – Firms do not affect market price $p_M$

  – Will firm produce at $p > p_M$?
  
  – Will firm produce at $p < p_M$?

  – $\implies p = p_M$
\begin{itemize}
  \item Revenue: \(py = pf(L, K)\)
  \item Cost: \(wL + rK\)
  \item Profit \(pf(L, K) - wL - rK\)
\end{itemize}
• Agent optimization:

$$\max_{L,K} pf (L, K) - wL - rK$$

• First order conditions:

$$pf'_L (L, K) - w = 0$$

and

$$pf'_K (L, K) - r = 0$$

• Second order conditions? $$pf''_{L,L} (L, K) < 0$$ and

$$|H| = \begin{vmatrix} pf''_{L,L} (L, K) & pf''_{L,K} (L, K) \\ pf''_{L,K} (L, K) & pf''_{K,K} (L, K) \end{vmatrix} =$$

$$= p^2 \left[ f''_{L,L} f''_{K,K} - (f''_{L,K})^2 \right] > 0$$

• Need $$f''_{L,K}$$ not too large for maximum
• Comparative statics with respect to $p$, $w$, and $r$.

• What happens if $w$ increases?

\[
\frac{\partial L^*}{\partial w} = -\frac{\begin{vmatrix} -1 & p f''_{L,K} (L, K) \\ 0 & p f''_{K,K} (L, K) \end{vmatrix}}{p f''_{L,L} (L, K) p f''_{L,K} (L, K) \begin{vmatrix} f''_{L,K} (L, K) & p f''_{K,K} (L, K) \\ p f''_{L,K} (L, K) & p f''_{K,K} (L, K) \end{vmatrix}} < 0
\]

and

\[
\frac{\partial L^*}{\partial r} =
\]

• Sign of $\partial L^*/\partial r$ depends on $f''_{L,K}$.
Introduction to Market Equilibrium

• Nicholson, Ch. 10, pp. 279–295 [OLD: Ch. 14, pp. 368–382.

• Two ways to analyze firm behavior:
  – Two-Step Cost Minimization
  – One-Step Profit Maximization

• What did we learn?
  – Optimal demand for inputs $L^*, K^*$ (see above)
  – Optimal quantity produced $y^*$
• **Supply function.** \( y = y^* (p, w, r) \)

  – From profit maximization:
    \[
    y = f \left( L^* (p, w, r), K^* (p, w, r) \right)
    \]

  – From cost minimization:
    \( MC \) curve above \( AC \)

  – Supply function is increasing in \( p \)

• Market Equilibrium. Equate demand and supply.

• Aggregation?

• Industry supply function!
5 Next Lecture

- Aggregation

- Market Equilibrium

- Comparative Statics of Equilibrium

- Taxes and Subsidies

- Long-Run Equilibrium