Outline

1. Supply Function

2. One-step Profit Maximization

3. Introduction to Market Equilibrium

4. Aggregation

5. Market Equilibrium in the Short-Run
1 Supply Function

- Supply function: \( y^* = y^*(w, r, p) \)

- What happens to \( y^* \) as \( p \) increases?

- Is the supply function upward sloping?

- Remember f.o.c:
  \[ p - c'_y (w, r, y) = 0 \]

- Implicit function:
  \[ \frac{\partial y^*}{\partial p} = - \frac{1}{-c''_{y,y} (w, r, y)} > 0 \]

  as long as s.o.c. is satisfied.

- Yes! Supply function is upward sloping.
2 One-step Profit Maximization

- Nicholson, Ch. 9, pp. 265–270 [OLD: Ch. 13, pp. 346–350].

- One-step procedure: maximize profits

- Perfect competition. Price $p$ is given
  
  - Firms are small relative to market
  
  - Firms do not affect market price $p_M$

  - Will firm produce at $p > p_M$?
  
  - Will firm produce at $p < p_M$?

  - $\implies p = p_M$
• Revenue: \( py = pf (L, K) \)

• Cost: \( wL + rK \)

• Profit \( pf (L, K) - wL - rK \)
• Agent optimization:
\[
\max_{L,K} pf(L, K) - wL - rK
\]

• First order conditions:
\[
pf_L'(L, K) - w = 0
\]

and
\[
pf_K'(L, K) - r = 0
\]

• Second order conditions? \( pf''_{L,L} (L, K) < 0 \) and
\[
|H| = \begin{vmatrix} pf''_{L,L} (L, K) & pf''_{L,K} (L, K) \\ pf''_{K,L} (L, K) & pf''_{K,K} (L, K) \end{vmatrix} = \\
p^2 \left[ pf''_{L,L} pf''_{K,K} - (pf''_{L,K})^2 \right] > 0
\]

• Need \( pf''_{L,K} \) not too large for maximum
• Comparative statics with respect to to $p$, $w$, and $r$.

• What happens if $w$ increases?

\[
\frac{\partial L^*}{\partial w} = -\frac{\begin{vmatrix} -1 & p f''_{L,K}(L,K) \\ 0 & p f''_{K,K}(L,K) \end{vmatrix}}{\begin{vmatrix} p f''_{L,L}(L,K) & p f''_{L,K}(L,K) \\ p f''_{L,K}(L,K) & p f''_{K,K}(L,K) \end{vmatrix}} < 0
\]

and

\[
\frac{\partial L^*}{\partial r} = \quad
\]

• Sign of $\partial L^*/\partial r$ depends on $f''_{L,K}$. 

3 Introduction to Market Equilibrium


- Two ways to analyze firm behavior:
  - Two-Step Cost Minimization
  - One-Step Profit Maximization

- What did we learn?
  - Optimal demand for inputs $L^*$, $K^*$ (see above)
  - Optimal quantity produced $y^*$
• Supply function. $y = y^* (p, w, r)$

  – From profit maximization:
    $$y = f (L^* (p, w, r), K^* (p, w, r))$$

  – From cost minimization:
    $$MC \text{ curve above } AC$$

  – Supply function is increasing in $p$

• Market Equilibrium. Equate demand and supply.

• Aggregation?

• Industry supply function!
4  Aggregation

4.1  Producers aggregation

- $J$ companies, $j = 1, \ldots, J$, producing good $i$

- Company $j$ has supply function

\[ y_i^j = y_i^{j*} (p_i, w, r) \]

- Industry supply function:

\[ Y_i (p_i, w, r) = \sum_{j=1}^J y_i^{j*} (p_i, w, r) \]

- Graphically,
4.2 Consumer aggregation

- Nicholson, Ch. 10, pp. 279–282 [OLD: Ch. 7, pp. 172–176]

- One-consumer economy

- Utility function $u(x_1, \ldots, x_n)$

- Prices $p_1, \ldots, p_n$

- Maximization $\Rightarrow$

  \[
  x_1^* = x_1^*(p_1, \ldots, p_n, M), \\
  \vdots \\
  x_n^* = x_n^*(p_1, \ldots, p_n, M).
  \]
• Focus on good $i$. Fix prices $p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n$ and $M$

• **Single-consumer demand function:**

$$x_i^* = x_i^* (p_i | p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n, M)$$

• What is sign of $\partial x_i^*/\partial p_i$?

• Negative if good $i$ is normal

• Negative or positive if good $i$ is inferior
• **Aggregation:** $J$ consumers, $j = 1, \ldots, J$

• Demand for good $i$ by consumer $j$:

\[
x_{i}^{j*} = x_{i}^{j*}(p_1, \ldots, p_n, M^j)
\]

• Market demand $X_i$:

\[
X_i\left(p_1, \ldots, p_n, M^1, \ldots, M^J\right) = \sum_{j=1}^{J} x_{i}^{j*}(p_1, \ldots, p_n, M^j)
\]

• Graphically,
• Notice: market demand function depends on distribution of income $M^J$

• Market demand function $X_i$:
  
  – Consumption of good $i$ as function of prices $p$
  
  – Consumption of good $i$ as function of income distribution $M^j$
5 Market Equilibrium in the Short-Run

- Nicholson, Ch. 14, pp. 368–382.

- What is equilibrium price $p_i$?

- Magic of the Market...

- Equilibrium: No excess supply, No excess demand

- Prices $p^*$ equates demand and supply of good $i$:

$$Y^* = Y^*_i (p^*_i, w, r) = X^D_i (p^*_1, ..., p^*_n, M^1, ..., M^J)$$
• Graphically,

• Notice: in short-run firms can make positive profits
• Comparative statics exercises with endogenous price $p_i$:
  
  – increase in wage $w$ or interest rate $r$:

  – change in income distribution
6 Comparative statics of equilibrium

• Supply and Demand function of parameter $\alpha$:

  $\text{\hspace{1cm}}$ $Y_i^S (p_i, w, r, \alpha)$
  $\text{\hspace{1cm}}$ $X_i^D (p, M, \alpha)$

• How does $\alpha$ affect $p^*$ and $Y^*$?

• Comparative statics with respect to $\alpha$

• Equilibrium:

  $Y_i^S (p_i, w, r, \alpha) = X_i^D (p, M, \alpha)$
• Can write equilibrium as implicit function:

\[ Y_i^S(p_i, w, r, \alpha) - X_i^D(p, M, \alpha) = 0 \]

• What is \( dp^*/d\alpha \)?

• Implicit function theorem:

\[
\frac{\partial p^*}{\partial \alpha} = -\frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p} - \frac{\partial Y^S}{\partial \alpha} - \frac{\partial X^D}{\partial \alpha}
\]

• What is sign of denominator?

• Sign of \( \partial p^*/\partial \alpha \) is negative of sign of numerator
• Examples:

1. **Fad.** Good becomes more fashionable: $\frac{\partial X^D}{\partial \alpha} > 0 \implies \frac{\partial p^*}{\partial \alpha} > 0$

2. **Recession in Europe.** Negative demand shock for US firms: $\frac{\partial X^D}{\partial \alpha} < 0 \implies \frac{\partial p^*}{\partial \alpha} < 0$

3. **Oil shock.** Import prices increase: $\frac{\partial Y^S}{\partial \alpha} < 0 \implies \frac{\partial p^*}{\partial \alpha} > 0$

4. **Computerization.** Improvement in technology. $\frac{\partial Y^S}{\partial \alpha} > 0 \implies \frac{\partial p^*}{\partial \alpha} < 0$
7 Next Lecture

- Elasticities

- Taxes and Subsidies

- Long-Run Equilibrium