Economics 101A
(Lecture 19)

Stefano DellaVigna

November 3, 2005
Outline

1. Comparative Statics of Equilibrium

2. Elasticities

3. Response to Taxes

4. Market Equilibrium in The Long-Run
1 Comparative statics of equilibrium

• Supply and Demand function of parameter $\alpha$:
  
  $\begin{align*}
  &\quad Y_i^S (p_i, w, r, \alpha) \\
  &\quad X_i^D (p, M, \alpha)
  \end{align*}$

• How does $\alpha$ affect $p^*$ and $Y^*$?

• Comparative statics with respect to $\alpha$

• Equilibrium:

  \[ Y_i^S (p_i, w, r, \alpha) = X_i^D (p, M, \alpha) \]
• Can write equilibrium as implicit function:

\[ Y_i^S (p_i, w, r, \alpha) - X_i^D (p, M, \alpha) = 0 \]

• What is \( dp^*/d\alpha \)?

• Implicit function theorem:

\[
\frac{\partial p^*}{\partial \alpha} = -\frac{\partial Y^S}{\partial p}\frac{\partial X^D}{\partial p} - \frac{\partial Y^S}{\partial \alpha}\frac{\partial X^D}{\partial \alpha}
\]

• What is sign of denominator?

• Sign of \( \partial p^*/\partial \alpha \) is negative of sign of numerator
• Examples:

1. **Fad.** Good becomes more fashionable: $\frac{\partial X^D}{\partial \alpha} > 0 \implies \frac{\partial p^*}{\partial \alpha} > 0$

2. **Recession in Europe.** Negative demand shock for US firms: $\frac{\partial X^D}{\partial \alpha} < 0 \implies \frac{\partial p^*}{\partial \alpha} < 0$

3. **Oil shock.** Import prices increase: $\frac{\partial Y^S}{\partial \alpha} < 0 \implies \frac{\partial p^*}{\partial \alpha} > 0$

4. **Computerization.** Improvement in technology. $\frac{\partial Y^S}{\partial \alpha} > 0 \implies \frac{\partial p^*}{\partial \alpha} < 0$
2 Elasticities

• [Not in midterm]

• Nicholson, Ch.1, pp. 27–28 [OLD: Ch.7, pp. 176–177]

• How do we interpret magnitudes of $\frac{\partial p^*/\partial \alpha}{\partial \alpha}$?

• Result depends on units of measure.

• Can we write $\frac{\partial p^*/\partial \alpha}{\partial \alpha}$ in a unit-free way?

• Yes! Use **elasticities**.

• Elasticity of $x$ with respect to parameter $p$ is

$$\varepsilon_{x,p} = \frac{\partial x p}{\partial p x}$$
• Interpretation: Percent response in \( x \) to percent change in \( p \):

\[
\varepsilon_{x,p} = \frac{\partial x}{\partial p} = \lim_{dp \to 0} \frac{x (p + dp) - x (p) p}{dp} = \lim_{dp \to 0} \frac{dx}{x} \frac{dx}{dp} \frac{dp}{p}
\]

where \( dx \equiv x (p + dp) - x (p) \).

• Now, show

\[
\varepsilon_{x,p} = \frac{\partial \ln x}{\partial \ln p}
\]

• Notice: This makes sense only for \( x > 0 \) and \( p > 0 \)
• Proof. Consider function

\[ x = f(p) \]

• Rewrite as

\[ \ln(x) = \ln f(p) = \ln f(e^{\ln(p)}) \]

• Define \( \hat{x} = \ln(x) \) and \( \hat{p} = \ln(p) \)

• This implies

\[ \hat{x} = \ln f(e^{\hat{p}}) \]

• Get

\[
\frac{\partial \hat{x}}{\partial \hat{p}} = \frac{\partial \ln x}{\partial \ln p} = \frac{1}{f(e^{\hat{p}})} \frac{\partial f(e^{\hat{p}})}{\partial \hat{p}} e^{\hat{p}} = \frac{\partial x}{\partial p} \frac{\partial p}{\partial x} \]
• Example with Cobb-Douglas utility function

\[ U(x, y) = x^\alpha y^{1-\alpha} \] implies solutions

\[ x^* = \frac{M}{p_x}, \quad y^* = (1 - \alpha) \frac{M}{p_y} \]

• Elasticity of demand with respect to own price \( \varepsilon_{x,p_x} \):

\[ \varepsilon_{x,p_x} = \frac{\partial x^* \cdot p_x}{\partial p_x \cdot x^*} = -\frac{\alpha M}{(p_x)^2} \cdot \frac{p_x}{\alpha M} = -1 \]

• Elasticity of demand with respect to other price \( \varepsilon_{x,p_y} = 0 \)
• Go back to problem above:

\[
\frac{\partial p^*}{\partial \alpha} = - \frac{\partial Y^S}{\partial \alpha} - \frac{\partial X^D}{\partial \alpha} - \frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p}
\]

• Use elasticities to rewrite response of \( p \) to change in \( \alpha \):

\[
\frac{\partial p^* \alpha}{\partial \alpha p} = - \left( \frac{\partial Y^S}{\partial \alpha} - \frac{\partial X^D}{\partial \alpha} \right) \frac{\alpha}{Y} \left( \frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p} \right) \frac{p}{Y}
\]

or (using fact that \( X^{D*} = Y^{s*} \))

\[
\varepsilon_{p,\alpha} = - \frac{\varepsilon_{S,\alpha} - \varepsilon_{D,\alpha}}{\varepsilon_{S,p} - \varepsilon_{D,p}}
\]

• We are likely to know elasticities from empirical studies.
3 Response to taxes

- Nicholson, Ch. 11, pp. 322–323 [OLD: Ch. 15, pp. 407–408]

- Per-unit tax $t$

- Write price $p_i$ as price including tax

- Supply: $Y_i^S (p_i - t, w, r)$

- Demand: $X_i^D (p, M)$

$$Y_i^S (p_i - t, w, r) - X_i^D (p, M) = 0$$

- What is $dp^*/dt$?
• Comparative statics:

\[
\frac{\partial p^*}{\partial t} = -\frac{\partial Y^S}{\partial t} = \frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p} = -\frac{\partial Y^S}{\partial p} \frac{p}{X} = \frac{\left(\frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p}\right) p}{X} = \frac{\varepsilon_{S,p}}{\varepsilon_{S,p} - \varepsilon_{D,p}}
\]

• How about price received by suppliers \( p^* - t \)?

\[
\frac{\partial (p^* - t)}{\partial t} = \frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p} - 1 = \frac{\varepsilon_{D,p}}{\varepsilon_{S,p} - \varepsilon_{D,p}}
\]
• *Inflexible Supply.* (Capacity is fixed) Supply curve vertical \( (\varepsilon_{S,p} = 0) \)

• Producers bear burden of tax

• *Flexible Supply.* (Constant Returns to Scale) Supply curve horizontal \( (\varepsilon_{S,p} \to \infty) \)

• Consumers bear burden of tax
• **Inflexible demand.** Demand curve vertical ($\varepsilon_{D,p} = 0$)?

• Consumers bear burden

• General lesson: Least elastic side bears larger part of burden

• What happens with a subsidy ($t < 0$)?

• What happens to quantity sold?

• Use demand curve:

$$\frac{\partial X^D_*}{\partial t} = \frac{\partial X^D_*}{\partial p^*} \frac{\partial p^*}{\partial t}$$

and use expression for $\partial p^*/\partial t$ above.
4 Market Equilibrium in the Long-Run

• Nicholson, Ch. 10, pp. 295–306 [OLD: Ch. 14, pp. 382–394]

• So far, short-run analysis: no. of firms fixed to $J$

• How about firm entry?

• Long-run: free entry of firms

• When do firms enter? When positive profits!

• This drives profits to zero.
• Entry of one firm on industry supply function $Y^S(p, w, r)$ from period $t - 1$ to period $t$:

$$Y^S_t(p, w, r) = Y^S_{t-1}(p, w, r) + y(p, w, r)$$
• Supply function shifts to right and flattens:

\[ Y_t^S(p, w, r) = Y_{t-1}^S(p, w, r) + y(p, w, r) \]

\[ > Y_{t-1}^S(p, w, r) \text{ for } p \text{ above } AC \]

since \( y(p, w, r) > 0 \) on the increasing part of the supply function.

• Also:

\[ Y_t^S(p, w, r) = Y_{t-1}^S(p, w, r) \text{ for } p \text{ below } AC \]

since for \( p \) below \( AC \) the firm does not produce \( (y(p, w, r) = 0) \).
• Flattening:

\[
\frac{\partial Y_t^S(p, w, r)}{\partial p} = \frac{\partial Y_{t-1}^S(p, w, r)}{\partial p} + \frac{\partial y(p, w, r)}{\partial p}
\]

\[
> \frac{\partial Y_{t-1}^S(p, w, r)}{\partial p}
\]

for \( p \) above \( AC \) since \( \frac{\partial y(p, w, r)}{\partial p} > 0 \).

• Also:

\[
\frac{\partial Y_t^S(p, w, r)}{\partial p} = \frac{\partial Y_{t-1}^S(p, w, r)}{\partial p}
\]

for \( p \) below \( AC \)

• Profits go down since demand curve downward-sloping
• In the long-run, price equals minimum of average cost

• Why? Entry of new firms as long as $\pi > 0$

• $(\pi > 0$ as long as $p > AC)$

• Entry of new firm until $\pi = 0 \Rightarrow$ entry until $p = AC$

• Also:

$$\text{If } C''(y) = \frac{C(y)}{y}, \text{ then } \frac{\partial C(y)}{\partial y} = 0$$
• Graphically,
• Special cases:

• **Constant cost industry**

• Cost function of each company does not depend on number of firms
• **Increasing cost industry**

• Cost function of each company increasing in no. of firms

• Ex.: congestion in labor markets
• Decreasing cost industry

• Cost function of each company decreasing in no. of firms

• Ex.: set up office to promote exports
5 Next Lecture

- Consumer and Producer Surplus

- Market Power

- Monopoly

- Price Discrimination