Outline

1. Game Theory II

2. Dynamic Games

3. Oligopoly: Stackelberg
1 Game Theory II

- Penalty kick in soccer (matching pennies)

<table>
<thead>
<tr>
<th>Kicker \ Goalie</th>
<th>L</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>0,1</td>
<td>1,0</td>
</tr>
<tr>
<td>R</td>
<td>1,0</td>
<td>0,1</td>
</tr>
</tbody>
</table>

- Equilibrium always exists in mixed strategies $\sigma$
• Mixed strategy: allow for probability distribution.

• Back to penalty kick:
  
  – Kicker kicks left with probability $k$
  
  – Goalie kicks left with probability $g$

  – utility for kicker of playing $L$:

$$U_K(L, \sigma) = gU_K(L, L) + (1 - g)U_K(L, R)$$
$$= (1 - g)$$

  – utility for kicker of playing $R$:

$$U_K(R, \sigma) = gU_K(R, L) + (1 - g)U_K(R, R)$$
$$= g$$
• Optimum?

  - $L \succ R$ if $1 - g > g$ or $g < 1/2$
  - $R \succ L$ if $1 - g < g$ or $g > 1/2$
  - $L \sim R$ if $1 - g = g$ or $g = 1/2$

• Plot best response for kicker

• Plot best response for goalie
• Nash Equilibrium is:
  – fixed point of best response correspondence

  – crossing of best response correspondences
2 Dynamic Games

• Nicholson, Ch. 15, pp. 449–454. [OLD: Ch. 10, pp. 256–259]

• Dynamic games: one player plays after the other

• Decision trees
  – Decision nodes
  – Strategy is a plan of action at each decision node
• Example: battle of the sexes game

<table>
<thead>
<tr>
<th></th>
<th>She \ He</th>
<th>Ballet</th>
<th>Football</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ballet</td>
<td>2, 1</td>
<td>0, 0</td>
<td></td>
</tr>
<tr>
<td>Football</td>
<td>0, 0</td>
<td>1, 2</td>
<td></td>
</tr>
</tbody>
</table>

• Dynamic version: she plays first
• **Subgame-perfect equilibrium.** At each node of the tree, the player chooses the strategy with the highest payoff, given the other players’ strategy

• Backward induction. Find optimal action in last period and then work backward

• Solution
• Example 2: Entry Game

<table>
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<th>Do not Enter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enter</td>
<td>$-1, -1$</td>
<td>$10, 0$</td>
</tr>
<tr>
<td>Do not Enter</td>
<td>$0, 5$</td>
<td>$0, 0$</td>
</tr>
</tbody>
</table>

• Exercise. Dynamic version.

• Coordination games solved if one player plays first
• Can use this to study finitely repeated games

• Suppose we play the prisoner's dilemma game ten times.

\[
\begin{array}{c|ccc}
1 \setminus 2 & D & ND \\
D & -4, -4 & -1, -5 \\
ND & -5, -1 & -2, -2 \\
\end{array}
\]

• What is the subgame perfect equilibrium?
3 Oligopoly: Stackelberg

- Setting as in problem set.

- 2 Firms

- Cost: \( c(y) = cy \), with \( c > 0 \)

- Demand: \( p(Y) = a - bY \), with \( a > c > 0 \) and \( b > 0 \)

- Difference: Firm 1 makes the quantity decision first

- Use subgame perfect equilibrium
- **Solution:**

- Solve first for Firm 2 decision as function of Firm 1 decision:

\[
\max_{y_2} (a - by_2 - by_1^*) y_2 - cy_2
\]

- F.o.c.:

\[
a - 2by_2^* - by_1^* - c = 0
\]

or

\[
y_2^* = \frac{a - c}{2b} - \frac{y_1^*}{2}.
\]

\[
p_D^* = a - bY_D^* = a - b \left( \frac{2\frac{a - c}{3b}}{3} \right) = \frac{1}{3}a + \frac{2}{3}c.
\]
• Firm 1 takes this response into account in the maximization:

$$\max_{y_1} (a - by_1 - by_2^* (y_1)) y_1 - cy_1$$

or

$$\max_{y_1} \left( a - by_1 - b \left( \frac{a - c}{2b} - \frac{y_1}{2} \right) \right) y_1 - cy_1$$

• F.o.c.:

$$a - 2by_1 - \frac{(a - c)}{2} + by_1 - c = 0$$

or

$$y_1^* = \frac{a - c}{2b}$$

and

$$y_2^* = \frac{a - c}{2b} - \frac{y_1^*}{2} = \frac{a - c}{2b} - \frac{a - c}{4b} = \frac{a - c}{4b}.$$
• Total production:

\[ Y_D^* = y_1^* + y_2^* = 3\frac{a - c}{4b} \]

• Price equals

\[ p^* = a - b \left( \frac{3a - c}{4b} \right) = \frac{1}{4}a + \frac{3}{4}c \]

• Compare to monopoly:

\[ y_M^* = \frac{a - c}{2b} \]

and

\[ p_M^* = \frac{a + c}{2}. \]

• Compare to Cournot:

\[ Y_D^* = y_1^* + y_2^* = 2\frac{a - c}{3b} \]

and

\[ p_D^* = \frac{1}{3}a + \frac{2}{3}c. \]
• Figure

• Compare with Cournot outcome
4 Next lecture

- General equilibrium
- Edgeworth Box