Economics 101A
(Lecture 9)

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Outline

1. Slutsky Equation II
2. Complements and substitutes
3. Do utility functions exist?
4. Application 1: Labor Supply
1 Slutsky Equation


- Slutsky Equation:

\[
\frac{\partial x^*_i (p, M)}{\partial p_i} = \frac{\partial h_i (p, v(p, M))}{\partial p_i} - x^*_i (p_1, p_2, M) \frac{\partial x^*_i (p, M)}{\partial M}
\]

- Important result! Allows decomposition into substitution and income effect
• Two effects of change in price:

1. Substitution effect negative: \( \frac{\partial h_i(p, v(p, M))}{\partial p_i} < 0 \)

2. Income effect: \( -x^*_1(p_1, p_2, M) \frac{\partial x^*_i(p, M)}{\partial M} \)
   - negative if good \( i \) is normal \( (\frac{\partial x^*_i(p, M)}{\partial M} > 0) \)
   - positive if good \( i \) is inferior \( (\frac{\partial x^*_i(p, M)}{\partial M} < 0) \)

• Overall, sign of \( \frac{\partial x^*_i(p, M)}{\partial p_i} \)?

  - negative if good \( i \) is normal
  - it depends if good \( i \) is inferior
• Example 1 (ctd.): Cobb-Douglas. Apply Slutsky equation

• $x_i^* = \alpha M/p_i$

• $h_i^* = \quad$

• Derivative of Hicksian demand with respect to price:

$$\frac{\partial h_i(p, \bar{u})}{\partial p_i} = \quad$$

• Rewrite $h_i^*$ as function of $m$: $h_i(p, v(p, M))$

• Compute $v(p, M) = \quad$
- Substitution effect:

\[
\frac{\partial h_i(p, v(p, M))}{\partial p_i} =
\]

- Income effect:

\[-x_i^*(p_1, p_2, M) \frac{\partial x_i^*(p, M)}{\partial M} =
\]

- Sum them up to get

\[
\frac{\partial x_i^*(p, M)}{\partial p_i} =
\]

- It works!
2 Complements and substitutes

- Nicholson, Ch. 6, pp. 161–166 [OLD: 152–158].

- How about if price of another good changes?

- Generalize Slutsky equation

- Slutsky Equation:

\[
\frac{\partial x^*_i(p, M)}{\partial p_j} = \frac{\partial h_i(p, v(p, M))}{\partial p_j} - x^*_j(p_1, p_2, M) \frac{\partial x^*_i(p, M)}{\partial M}
\]
• Substitution effect

\[ \frac{\partial h_i(p, v(p, M))}{\partial p_j} > 0 \]

for \( n = 2 \) (two goods). Ambiguous for \( n > 2 \).

• Income effect:

\[ -x_j^*(p_1, p_2, M) \frac{\partial x_i^*(p, M)}{\partial M} \]

– negative if good \( i \) is normal

– positive if good \( i \) is inferior

• How do we define complements and substitutes?
• Def. 1. Goods $i$ and $j$ are \textbf{gross substitutes} at price $p$ and income $M$ if

$$\frac{\partial x_i^*(p, M)}{\partial p_j} > 0$$

• Def. 2. Goods $i$ and $j$ are \textbf{gross complements} at price $p$ and income $M$ if

$$\frac{\partial x_i^*(p, M)}{\partial p_j} < 0$$

• Example 1 (ctd.): $x_1^* = \alpha M/p_1$, $x_2^* = \beta M/p_2$.

• Gross complements or gross substitutes? Neither!

• Notice: $\frac{\partial x_i^*(p, M)}{\partial p_j}$ is usually different from $\frac{\partial x_j^*(p, M)}{\partial p_i}$
• Better definition.

• Def. 3. Goods \( i \) and \( j \) are **net substitutes** at price \( p \) and income \( M \) if

\[
\frac{\partial h^*_i(p, v(p, M))}{\partial p_j} = \frac{\partial h^*_j(p, v(p, M))}{\partial p_i} > 0
\]

• Def. 4. Goods \( i \) and \( j \) are **net complements** at price \( p \) and income \( M \) if

\[
\frac{\partial h^*_i(p, v(p, M))}{\partial p_j} = \frac{\partial h^*_j(p, v(p, M))}{\partial p_i} < 0
\]

• Example 1 (ctd.): \( h^*_1 = \bar{u} \left( \frac{\alpha p_2}{1-\alpha p_1} \right)^{1-\alpha} \)

• Net complements or net substitutes? Net substitutes!
3 Do utility functions exist?

- Preferences and utilities are theoretical objects

- Many different ways to write them

- How do we tie them to the world?

- Use actual choices – revealed preferences approach
• Typical economists’ approach. Compromise of:
  – realism
  – simplicity

• Assume a class of utility functions (CES, Cobb-Douglas...) with free parameters

• Estimate the parameters using the data
4 Labor Supply

- Nicholson Ch. 16, pp. 477–484 [OLD: Ch. 22, pp. 606–613.]

- Labor supply decision: how much to work in a day.

- Goods: consumption good $c$, hours worked $h$

- Price of good $p$, hourly wage $w$

- Consumer spends $24 - h = l$ hours in units of leisure

- Utilify function: $u(c, l)$
• Budget constraint?

• Income of consumer: $M + wh = M + w(24 - l)$

• Budget constraint: $pc \leq M + w(24 - l)$ or
  
  $$pc + wl \leq M + 24w$$

• Notice: leisure $l$ is a consumption good with price $w$. Why?

• General category: **opportunity cost**

• Instead of enjoying one hour of TV, I could have worked one hour and gained wage $w$.

• You should value the marginal hour of TV $w$!
• Opportunity costs are very important!

• Example 2. CostCo has a warehouse in SoMa

• SoMa used to have low cost land, adequate for warehouses

• Price of land in SoMa triples in 10 years.

• Should firm relocate the warehouse?
• Did costs of staying in SoMa go up?

• No.

• Did the opportunity cost of staying in SoMa go up?

• Yes!

• Firm can sell at high price and purchase land in cheaper area.
• Let’s go back to labor supply

• Maximization problem is

\[
\max u(c, l) \\
\text{s.t. } pc + wl \leq M + 24w
\]

• Standard problem (except for \(24w\))

• First order conditions

• Assume utility function Cobb-Douglas:

\[u(c, l) = c^\alpha l^{1-\alpha}\]
• Solution is

\[ c^* = \frac{\alpha M + 24w}{p} \]

\[ l^* = (1 - \alpha) \left( 24 + \frac{M}{w} \right) \]

• Both \( c \) and \( l \) are normal goods

• Unlike in standard Cobb-Douglas problems, \( c^* \) depends on price of other good \( w \)

• Why? Agents are endowed with \( M \) AND 24 hours of \( l \) in this economy

• Normally, agents are only endowed with \( M \)
5 Next Lectures

• More applications:
  – Intertemporal choice
  – Economics of Altruism