Economics 101A
(Lecture 11)

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Outline

1. Altruism and charitable donations II

2. Introduction to probability

3. Expected Utility

4. Risk Aversion
1 Altruism and Charitable Donations II

- Wendy computes the utility of Mark as a function of the donation $D$

- Mark maximizes

$$\max_{c_M} u(c_M)$$

subject to $c_M \leq M_M + D$

- Solution: $c_M^* = M_M + D$

- Wendy maximizes

$$\max_{c_M, D} u(c_W) + \alpha u(M_M + D)$$

subject to $c_W \leq M_W - D$
Rewrite as:
\[ \max_D u(M_W - D) + \alpha u (M_M + D) \]

First order condition:
\[ -u'(M_W - D^*) + \alpha u' (M_M + D^*) = 0 \]

Second order conditions:
\[ u''(M_W - D^*) + \alpha u'' (M_M + D^*) < 0 \]
• Assume $\alpha = 1$.

  – Solution?

  – \[ u'(M_W - D) = u'(M_M + D^*) \]

  – \[ M_W - D^* = M_M + D^* \] or \[ D^* = (M_W - M_M) / 2 \]

  – Transfer money so as to equate incomes!

  – Careful: $D < 0$ (negative donation!) if $M_M > M_W$

• Corrected maximization:

\[
\max_D u(M_W - D) + \alpha u(M_M + D) \\
\text{s.t.} \quad D \geq 0
\]

• Solution ($\alpha = 1$):

\[
D^* = \begin{cases} 
(M_W - M_M) / 2 & \text{if } M_W - M_M > 0 \\
0 & \text{otherwise}
\end{cases}
\]
• Assume interior solution. \((D^* > 0)\)

• Comparative statics 1 (altruism):

\[
\frac{\partial D^*}{\partial \alpha} = -\frac{u'(M_M + D^*)}{u''(M_W - D^*) + \alpha u''(M_M + D^*)} > 0
\]

• Comparative statics 2 (income of donor):

\[
\frac{\partial D^*}{\partial M_W} = -\frac{-u''(M_W + D^*)}{u''(M_W - D^*) + \alpha u''(M_M + D^*)} > 0
\]

• Comparative statics 3 (income of recipient):

\[
\frac{\partial D^*}{\partial M_M} = -\frac{\alpha u''(M_M + D^*)}{u''(M_W - D^*) + \alpha u''(M_M + D^*)} < 0
\]
• A quick look at the evidence

• From Andreoni (2002)
2 Introduction to Probability

• So far deterministic world:
  – income given, known $M$
  – interest rate known $r$

• But some variables are unknown at time of decision:
  – future income $M_1$?
  – future interest rate $r_1$?

• Generalize framework to allow for uncertainty
  – Events that are truly unpredictable (weather)
  – Event that are very hard to predict (future income)
• Probability is the language of uncertainty

• Example:
  
  - Income $M_1$ at $t = 1$ depends on state of the economy
  
  - Recession ($M_1 = 20$), Slow growth ($M_2 = 25$), Boom ($M_3 = 30$)
  
  - Three probabilities: $p_1$, $p_2$, $p_3$
  
  - $p_1 = P(M_1) = P($recession$)$

• Properties:
  
  - $0 \leq p_i \leq 1$
  
  - $p_1 + p_2 + p_3 = 1$
• Mean income: \( EM = \sum_{i=1}^{3} p_i M_i \)

• If \((p_1, p_2, p_3) = (1/3, 1/3, 1/3)\),
  \[
  EM = \frac{1}{3}20 + \frac{1}{3}25 + \frac{1}{3}30 = \frac{75}{3} = 25
  \]

• Variance of income: \( V(M) = \sum_{i=1}^{3} p_i (M_i - EM)^2 \)

• If \((p_1, p_2, p_3) = (1/3, 1/3, 1/3)\),
  \[
  V(M) = \frac{1}{3} (20 - 25)^2 + \frac{1}{3} (25 - 25)^2 + \frac{1}{3} (30 - 25)^2
  \]
  \[
  = \frac{1}{3} 5^2 + \frac{1}{3} 5^2 = 2/3 * 25
  \]

• Mean and variance if \((p_1, p_2, p_3) = (1/4, 1/2, 1/4)\)?
3 Expected Utility

- Nicholson, Ch. 18, pp. 533–541 [OLD: Ch. 8, pp. 198–206]

- Consumer at time 0 asks: what is utility in time 1?

- At \( t = 1 \) consumer maximizes

\[
\max U(c^1) \\
\text{s.t. } c^1_i \leq M^1_i + (1 + r)(M^0 - c^0)
\]

with \( i = 1, 2, 3 \).

- What is utility at optimum at \( t = 1 \) if \( U' > 0 \)?

- Assume for now \( M^0 - c^0 = 0 \)

- Utility \( U(M^1_i) \)

- This is uncertain, depends on which \( i \) is realized!
• How do we evaluate future uncertain utility?

• **Expected utility**

\[
EU = \sum_{i=1}^{3} p_i U \left( M_i^1 \right)
\]

• In example:

\[
EU = \frac{1}{3} U(20) + \frac{1}{3} U(25) + \frac{1}{3} U(30)
\]

• Compare with \( U(EC) = U(25) \).

• Agents prefer riskless outcome \( EM \) to uncertain outcome \( M \) if

\[
\frac{1}{3} U(20) + \frac{1}{3} U(25) + \frac{1}{3} U(30) < U(25) \quad \text{or} \quad \frac{1}{3} U(20) + \frac{1}{3} U(30) < \frac{2}{3} U(25) \quad \text{or} \quad \frac{1}{2} U(20) + \frac{1}{2} U(30) < U(25)
\]
• Picture
• Depends on sign of \( U'' \), on concavity/convexity

• Three cases:

  - \( U''(x) = 0 \) for all \( x \). (linearity of \( U \))
    \[
    * U(x) = a + bx
    \]
    \[
    * 1/2U(20) + 1/2U(30) = U(25)
    \]

  - \( U''(x) < 0 \) for all \( x \). (concavity of \( U \))
    \[
    * 1/2U(20) + 1/2U(30) < U(25)
    \]

  - \( U''(x) > 0 \) for all \( x \). (convexity of \( U \))
    \[
    * 1/2U(20) + 1/2U(30) > U(25)
    \]
• If $U''(x) = 0$ (linearity), consumer is indifferent to uncertainty

• If $U''(x) < 0$ (concavity), consumer dislikes uncertainty

• If $U''(x) > 0$ (convexity), consumer likes uncertainty

• Do consumers like uncertainty?

• Do you like uncertainty?
• **Theorem. (Jensen’s inequality)** If a function $f(x)$ is concave, the following inequality holds:

$$f(Ex) \geq Ef(x)$$

where $E$ indicates expectation. If $f$ is strictly concave, we obtain

$$f(Ex) > Ef(x)$$

• Apply to utility function $U$.

• Individuals dislike uncertainty:

$$U(Ex) \geq EU(x)$$

• Jensen’s inequality then implies $U$ concave ($U'' \leq 0$)

• Relate to diminishing marginal utility of income
4 Risk aversion

- Nicholson, Ch. 18, pp. 535–541 [OLD: Ch. 8, pp. 200–206].

- Risk aversion:
  - individuals dislike uncertainty
  - \( u \) concave, \( u'' < 0 \)

- Implications?
  - purchase of insurance (possible accident)
  - investment in risky asset (risky investment)
  - choice over time (future income uncertain)
• Experiment — Are you risk-averse?
5 Next Lectures

• Coefficient of risk aversion

• Applications:
  – Insurance
  – Portfolio choice
  – Consumption choice II