Economics 101A
(Lecture 12)

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Outline

1. Nobel Prize winners

2. Risk Aversion and Lottery

3. Insurance

4. Investment in Risky Asset

5. Measures of Risk Aversion

6. Mid-Term Feedback
1 Nobel Prize winner

• Edmund Phelps (Columbia University)

• Macroeconomist – You get to hear about him in 101B

• Contribution:
  – Price setting should account for price expectations
  – You cannot keep raising money supply to lower unemployment
  – People will come to expect the price increase

• Also: Model in Phelps and Pollak (1968) – Antecedent of self-control models (next lecture)
2 Risk Aversion and Lottery

- Are you risk-averse?

- Let’s see...
3 Insurance

- Nicholson, Ch. 18, pp. 545–551 [OLD: Ch. 8, pp. 211-216] Notice: different treatment than in class

- Individual has:
  - wealth $w$
  - utility function $u$, with $u' > 0$, $u'' < 0$

- Probability $p$ of accident with loss $L$

- Insurance offers coverage:
  - premium $q$ for each $1$ paid in case of accident
  - units of coverage purchased $\alpha$
• Individual maximization:

\[
\max_{\alpha} (1 - p) u (w - q\alpha) + pu (w - q\alpha - L + \alpha) \\
\text{s.t.} \alpha \geq 0
\]

• Assume \( \alpha^* \geq 0 \), check later

• First order conditions:

\[
0 = -q (1 - p) u' (w - q\alpha) \\
+ (1 - q) pu' (w - q\alpha - L + \alpha)
\]

or

\[
\frac{u' (w - q\alpha)}{u' (w - q\alpha - L + \alpha)} = \frac{1 - q}{q} \frac{p}{1 - p}.
\]

• Assume first \( q = p \) (insurance is fair)

• Solution for \( \alpha^* =? \)
• $\alpha^* > 0$, so we are ok!

• What if $q > p$ (insurance needs to cover operating costs)?

• Insurance will be only partial (if at all)

• Exercise: Check second order conditions!
4 Investment in Risk Asset

- Individual has:
  - wealth \( w \)
  - utility function \( u \), with \( u' > 0 \)

- Two possible investments:
  - Asset B (bond) yields return 1 for each dollar
  - Asset S (stock) yields uncertain return \((1 + r)\):
    * \( r = r_+ > 0 \) with probability \( p \)
    * \( r = r_- < 0 \) with probability \( 1 - p \)
    * \( Er = pr_+ + (1 - p) r_- > 0 \)

- Share of wealth invested in stock \( S = \alpha \)
- Individual maximization:

\[
\max_{\alpha} (1 - p) \, u(w[ (1 - \alpha) + \alpha (1 + r_-)]) + \\
+ pu(w[ (1 - \alpha) + \alpha (1 + r_+)])
\]

\[
s.t. 0 \leq \alpha \leq 1
\]

- Case of risk neutrality: \( u(x) = a + bx, \ b > 0 \)

- Assume \( a = 0 \) (no loss of generality)

- Maximization becomes

\[
\max_{\alpha} b (1 - p) (w[1 + \alpha r_-]) + bp (w[1 + \alpha r_+])
\]

or

\[
\max_{\alpha} bw + \alpha bw [(1 - p) r_- + pr_+]
\]

- Sign of term in square brackets? Positive!

- Set \( \alpha^* = 1 \)
• Case of risk aversion: \( u'' < 0 \)

• Assume \( 0 \leq \alpha^* \leq 1 \), check later

• First order conditions:

\[
0 = (1 - p) (wr_-) u'(w [1 + \alpha r_-]) + \\
+ p (wr_+) u'(w [1 + \alpha r_+])
\]

• Can \( \alpha^* = 0 \) be solution?

• Solution is \( \alpha^* > 0 \) (positive investment in stock)

• Exercise: Check s.o.c.
5 Measures of Risk Aversion

- Nicholson, Ch. 18, pp. 541–545 [OLD: Ch. 8, pp. 207–210].

- How risk averse is an individual?

- Two measures:
  
  - Absolute Risk Aversion $r_A$:
    \[
    r_A = - \frac{u''(x)}{u'(x)}
    \]
  
  - Relative Risk Aversion $r_R$:
    \[
    r_R = - \frac{u''(x)}{u'(x)} x
    \]

- Examples in the Problem Set
6 Mid-Term Feedback

• Thanks for the feedback!
7 Next lecture and beyond

- **Tu:**
  - Time consistency
  - Time Inconsistency
  - Application to health clubs

- **Then:**
  - Begin Production
  - Returns to scale
  - Cost minimization