Outline

1. Mid-Term Feedback

2. Investment in Risky Asset II

3. Measures of Risk Aversion

4. Time Consistency

5. Time Inconsistency
1 Mid-Term Feedback

- Thanks for the feedback!
2 Investment in Risk Asset II

• Individual maximization:

\[
\max_{\alpha} (1 - p) \, u(w [(1 - \alpha) + \alpha (1 + r_-)]) + \\
+ p u(w [(1 - \alpha) + \alpha (1 + r_+)])
\]

s.t. \(0 \leq \alpha \leq 1\)

• Case of risk aversion: \(u'' < 0\)

• Assume \(0 \leq \alpha^* \leq 1\), check later

• First order conditions:

\[
0 = (1 - p) (wr_-) \, u'(w [1 + \alpha r_-]) + \\
+ p (wr_+) \, u'(w [1 + \alpha r_+])
\]

• Solution is \(\alpha^* > 0\) (positive investment in stock)

• Exercise: Check s.o.c.
3 Measures of Risk Aversion

- Nicholson, Ch. 18, pp. 541–545 [OLD: Ch. 8, pp. 207–210].

- How risk averse is an individual?

- Two measures:
  
  - Absolute Risk Aversion $r_A$:
    
    $$ r_A = -\frac{u''(x)}{u'(x)} $$
  
  - Relative Risk Aversion $r_R$:
    
    $$ r_R = -\frac{u''(x)}{u'(x)} x $$

- Examples in the Problem Set
4 Time consistency

• Intertemporal choice

• Three periods, $t = 0$, $t = 1$, and $t = 2$

• At each period $i$, agents:
  
  – have income $M_i' = M_i + \text{savings/debts from previous period}$
  
  – choose consumption $c_i$;
  
  – can save/borrow $M_i' - c_i$
  
  – no borrowing in last period: at $t = 2$ $M_2' = c_2$
• Utility function at $t = 0$

$$u(c_0, c_1, c_2) = U(c_0) + \frac{1}{1 + \delta}EU(c_1) + \frac{1}{(1 + \delta)^2}EU(c_2)$$

• Utility function at $t = 1$

$$u(c_1, c_2) = U(c_1) + \frac{1}{1 + \delta}EU(c_2)$$

• Utility function at $t = 2$

$$u(c_2) = U(c_2)$$

• $U' > 0$, $U'' < 0$
• Question: Do preferences of agent in period 0 agree with preferences of agent in period 1?

• Period 1.

• Budget constraint at $t = 1$:

$$c_1 + \frac{1}{1 + r} c_2 \leq M'_1 + \frac{1}{1 + r} M_2$$

• Maximization problem:

$$\max U(c_1) + \frac{1}{1 + \delta} EU(c_2)$$

s.t. $c_1 + \frac{1}{1 + r} c_2 \leq M'_1 + \frac{1}{1 + r} M_2$

• First order conditions:

• Ratio of f.o.c.s:

$$\frac{U'(c_1)}{EU'(c_2)} = \frac{1 + r}{1 + \delta}$$
• Back to period 0.

• Agent at time 0 can commit to consumption at time 1 as function of uncertain income $M_1$.

• Anticipated budget constraint at $t = 1$:

$$c_1 + \frac{1}{1+r}c_2 \leq M'_1 + \frac{1}{1+r}M_2$$

• Maximization problem:

$$\max U(c_0) + \frac{1}{1+\delta}U(c_1) + \frac{1}{(1+\delta)^2}EU(c_2)$$

$$s.t. \ c_1 + \frac{1}{1+r}c_2 \leq M'_1 + \frac{1}{1+r}M_2$$

• First order conditions:

• Ratio of f.o.c.s:

$$\frac{U''(c_1)}{EU''(c_2)} = \frac{1+r}{1+\delta}$$
• The two conditions coincide!

• **Time consistency.** Plans for future coincide with future actions.

• To see why, rewrite utility function $u(c_0, c_1, c_2)$:

$$U(c_0) + \frac{1}{1 + \delta}U(c_1) + \frac{1}{(1 + \delta)^2}EU(c_2)$$

$$= U(c_0) + \frac{1}{1 + \delta} \left[ U(c_1) + \frac{1}{1 + \delta}EU(c_2) \right]$$

• Expression in brackets coincides with utility at $t = 1$

• Is time consistency right?

  – addictive products (alcohol, drugs);

  – good actions (exercising, helping friends);

  – immediate gratification (shopping, credit card borrowing)
5 Time Inconsistency

- Alternative specification (Akerlof, 1991; Laibson, 1997; O’Donoghue and Rabin, 1999)

- Utility at time $t$ is $u(c_t, c_{t+1}, c_{t+2})$:

$$u(c_t) + \frac{\beta}{1 + \delta} u(c_{t+1}) + \frac{\beta}{(1 + \delta)^2} u(c_{t+2}) + \ldots$$

- Discount factor is

$$1, \frac{\beta}{1 + \delta}, \frac{\beta}{(1 + \delta)^2}, \frac{\beta}{(1 + \delta)^3}, \ldots$$

instead of

$$1, \frac{1}{1 + \delta}, \frac{1}{(1 + \delta)^2}, \frac{1}{(1 + \delta)^3}, \ldots$$

- What is the difference?

- Immediate gratification: $\beta < 1$
• Back to our problem: **Period 1**.

• Maximization problem:

\[
\max U(c_1) + \frac{\beta}{1 + \delta} EU(c_2) \\
\text{s.t. } c_1 + \frac{1}{1 + r} c_2 \leq M'_1 + \frac{1}{1 + r} M_2
\]

• First order conditions:

• Ratio of f.o.c.s:

\[
\frac{U'(c_1^*)}{EU'(c_2^*)} = \beta \frac{1 + r}{1 + \delta}
\]
• Now, period 0 with commitment.

• Maximization problem:

$$\max U(c_0) + \frac{\beta}{1 + \delta} U(c_1) + \frac{\beta}{(1 + \delta)^2} EU(c_2)$$

$$s.t. \ c_1 + \frac{1}{1 + r} c_2 \leq M_1' + \frac{1}{1 + r} M_2$$

• First order conditions:

• Ratio of f.o.c.s:

$$\frac{U'(c_1^*,c)}{EU'(c_2^*,c)} = \frac{1 + r}{1 + \delta}$$

• The two conditions differ!

• Time inconsistency: $c_1^{*,c} < c_1^*$ and $c_2^{*,c} > c_2^*$

• The agent allows him/herself too much immediate consumption and saves too little
• Ok, we agree. but should we study this as economists?

• YES!
  – One trillion dollars in credit card debt;
  – Most debt is in teaser rates;
  – Two thirds of Americans are overweight or obese;
  – $10bn health-club industry

• Is this testable?
  – In the laboratory?
  – In the field?
6 Next Lecture

- An Example: Health club Attendance
- Cost Minimization
- Solve an Example
- Cases in which s.o.c. are not satisfied
- Start Profit Maximization