Outline

1. Time Inconsistency II

2. Health Club Attendance

3. Production: Introduction

4. Production Function

5. Returns to Scale

6. Two-step Cost Minimization
1 Time Inconsistency II

- Alternative specification (Akerlof, 1991; Laibson, 1997; O’Donoghue and Rabin, 1999)

- Utility at time $t$ is $u(c_t, c_{t+1}, c_{t+2})$:
  
  $$u(c_t) + \frac{\beta}{1 + \delta} u(c_{t+1}) + \frac{\beta}{(1 + \delta)^2} u(c_{t+2}) + ...$$

- Discount factor is
  
  $$1, \frac{\beta}{1 + \delta}, \frac{\beta}{(1 + \delta)^2}, \frac{\beta}{(1 + \delta)^3}, ...$$

  instead of
  
  $$1, \frac{1}{1 + \delta}, \frac{1}{(1 + \delta)^2}, \frac{1}{(1 + \delta)^3}, ...$$

- What is the difference?

- Immediate gratification: $\beta < 1$
• Back to our problem: **Period 1.**

• Maximization problem:

\[
\begin{align*}
\max U(c_1) + \frac{\beta}{1 + \delta} EU(c_2) \\
\text{s.t. } c_1 + \frac{1}{1 + r} c_2 & \leq M'_1 + \frac{1}{1 + r} M_2
\end{align*}
\]

• First order conditions:

• Ratio of f.o.c.s:

\[
\frac{U'(c_1^*)}{EU'(c_2^*)} = \beta \frac{1 + r}{1 + \delta}
\]
• Now, **period 0** with commitment.

• Maximization problem:

$$\max U(c_0) + \frac{\beta}{1 + \delta} U(c_1) + \frac{\beta}{(1 + \delta)^2} EU(c_2)$$

subject to:

$$c_1 + \frac{1}{1 + r} c_2 \leq M_1' + \frac{1}{1 + r} M_2$$

• First order conditions:

• Ratio of f.o.c.s:

$$\frac{U'(c_{1*},c)}{EU'(c_{2*},c)} = \frac{1 + r}{1 + \delta}$$

• The two conditions differ!

• Time inconsistency: $c_{1*} < c_1^*$ and $c_{2*} > c_2^*$

• The agent allows him/herself too much immediate consumption and saves too little
• Ok, we agree. but should we study this as economists?

• YES!
  
  – One trillion dollars in credit card debt;
  
  – Most debt is in teaser rates;
  
  – Two thirds of Americans are overweight or obese;
  
  – $10bn health-club industry

• Is this testable?
  
  – In the laboratory?
  
  – In the field?
2 Health Club Attendance

• Health club industry study (DellaVigna and Malmendier, 2002)

• 3 health clubs

• Data on attendance from swiping cards

• Choice of contracts:
  – Monthly contract with average price of $75
  – 10-visit pass for $100

• Consider users that choose monthly contract. Attendance?
• Attend on average 4.8 times per month

• Pay on average over $17

• Average delay of 2.2 months ($185) between last attendance and contract termination

• Over membership, user could have saved $700 by paying per visit
• Health club attendance:
  
  – immediate cost \( c \)
  
  – delayed benefit \( b \)

• At sign-up (attend tomorrow):

\[
NB^t = -\frac{\beta}{1 + \delta}c + \frac{\beta}{(1 + \delta)^2}b
\]

• Plan to attend if \( NB^t > 0 \)

\[
c < \frac{1}{(1 + \delta)}b
\]
• Once moment to attend comes:

\[ NB = -c + \frac{\beta}{(1 + \delta)} b \]

• Attend if \( NB > 0 \)

\[ c < \frac{\beta}{(1 + \delta)} b \]
• Interpretations?

• Users are buying a commitment device

• User underestimate their future self-control problems:
  – They overestimate future attendance
  – They delay cancellation
3 Production: Introduction

• Second half of the economy. Production

• Example. Ford and the Minivan (Petrin, 2002):
  – Ford had idea: "Mini/Max" (early '70s)
  – Did Ford produce it?
    – No!
  – Ford was worried of cannibalizing station wagon sector
  – Chrysler introduces Dodge Caravan (1984)
  – Chrysler: $1.5bn profits (by 1987)!
• Why need separate treatment?

• Perhaps firms maximize utility...

• ...we can be more precise:
  – Competition
  – Institutional structure
4 Production Function


- Production function: \( y = f(z) \). Function \( f : \mathbb{R}^n_+ \rightarrow \mathbb{R}_+ \)

- Inputs \( z = (z_1, z_2, \ldots, z_n) \): labor, capital, land, human capital

- Output \( y \): Minivan, Intel Pentium III, mangoes (Philippines)

- Properties of \( f \):
  - no free lunches: \( f(0) = 0 \)
  - positive marginal productivity: \( f'_i(z) > 0 \)
  - decreasing marginal productivity: \( f''_{ii}(z) < 0 \)
• Isoquants $Q(y) = \{x | f(x) = y\}$

• Set of inputs $z$ required to produce quantity $y$

• Special case. Two inputs:
  
  - $z_1 = L$ (labor)
  
  - $z_2 = K$ (capital)

• Isoquant: $f(L, K) - y = 0$

• Slope of isoquant $dK/dL = MRTS$
• Convex production function if convex isoquants

• Reasonable: combine two technologies and do better!

• Mathematically, \( \frac{d^2K}{d^2L} = \)
5 Returns to Scale

- Nicholson, Ch. 7, pp. 190–193 [OLD: Ch. 11, pp. 275–278]

- Effect of increase in labor: $f'_L$

- Increase of all inputs: $f(tz)$ with $t$ scalar, $t > 1$

- How much does input increase?
  - Decreasing returns to scale: for all $z$ and $t > 1$, 
    \[ f(tz) < tf(z) \]
  - Constant returns to scale: for all $z$ and $t > 1$, 
    \[ f(tz) = tf(z) \]
- Increasing returns to scale: for all $z$ and $t > 1$,

$$f(tz) > tf(z)$$
• Example: \( y = f(K, L) = AK^\alpha L^\beta \)

• Marginal product of labor: \( f'_L = \)

• Decreasing marginal product of labor: \( f''_L = \)

• \( MRTS = \)

• Convex isoquant?

• Returns to scale: \( f(tK, tL) = A(tK)^\alpha (tL)^\beta = t^{\alpha+\beta} AK^\alpha L^\beta = t^{\alpha+\beta} f(K, L) \)
6 Two-step Cost minimization

• Nicholson, pp. 212–220 [OLD, Ch. 12, pp. 298–307]

• Objective of firm: Produce output that generates maximal profit.

• Decompose problem in two:
  – Given production level $y$, choose cost-minimizing combinations of inputs
  – Choose optimal level of $y$. 
7 Next Lecture

• Continue Cost Minimization

• Solve an Example

• Cases in which s.o.c. are not satisfied

• Start Profit Maximization