Outline

1. Cost Minimization: Example II

2. Cost Curves and Supply Function

3. One-step Profit Maximization

4. Introduction to Market Equilibrium
1 Cost Minimization: Example II

- Continue example above: \( y = f(L, K) = AK^\alpha L^\beta \)

- Define \( B := w \left( \frac{w \alpha}{r \beta} \right)^{-\frac{\alpha}{\alpha+\beta}} + r \left( \frac{w \alpha}{r \beta} \right)^{\frac{\beta}{\alpha+\beta}} \)

- Cost-minimizing output choice:

\[
\max p y - B \left( \frac{y}{A} \right)^{\frac{1}{\alpha+\beta}}
\]

- First order condition:

\[
p - \frac{1}{\alpha + \beta} \frac{B}{A} \left( \frac{y}{A} \right)^{\frac{1-(\alpha+\beta)}{\alpha+\beta}} = 0
\]

- Second order condition:

\[
- \frac{1}{\alpha + \beta} \frac{1 - (\alpha + \beta) B}{A^2} \left( \frac{y}{A} \right)^{\frac{1-2(\alpha+\beta)}{\alpha+\beta}} < 0
\]
• Solution:

- $\alpha + \beta > 1$ (IRS):
  
  * S.o.c. positive
  
  * Solution of f.o.c. is a minimum!
  
  * Solution is $y^* \to \infty$.
  
  * Keep increasing production since higher production is associated with higher returns
- $\alpha + \beta < 1$ (DRS):

  * s.o.c. negative. OK!

  * Solution of f.o.c. is an interior optimum

  * This is the only "well-behaved" case under perfect competition

  * Here can define a supply function
2 Cost Curves

• Nicholson, Ch. 8, pp. 220–228; Ch. 9, pp. 256–259 [OLD: Ch. 12, pp. 307–312 and Ch. 13, pp. 342–346.]

• Marginal costs $MC = \frac{\partial c}{\partial y} \rightarrow$ Cost minimization
  
  $$p = MC = \frac{\partial c (w, r, y)}{\partial y}$$

• Average costs $AC = \frac{c}{y} \rightarrow$ Does firm break even?
  
  $$\pi = py - c (w, r, y) > 0 \text{ iff }$$
  $$\frac{\pi}{y} = p - \frac{c (w, r, y)}{y} > 0 \text{ iff }$$
  $$\frac{c (w, r, y)}{y} = AC < p$$

• Supply function. Portion of marginal cost $MC$ above average costs.(price equals marginal cost)
• Assume only 1 input (expenditure minimization is trivial)

• **Case 1.** Production function. \( y = L^\alpha \)
  
  – Cost function? (cost of input is \( w \)):
  
  \[
  c(w, y) = wL^*(w, y) = wy^{1/\alpha}
  \]

  – Marginal cost?
  
  \[
  \frac{\partial c(w, y)}{\partial y} = \frac{1}{\alpha}wy^{(1-\alpha)/\alpha}
  \]

  – Average cost \( c(w, y)/y \)?
  
  \[
  \frac{c(w, y)}{y} = \frac{wy^{1/\alpha}}{y} = wy^{(1-\alpha)/\alpha}
  \]
• **Case 1a.** \( \alpha > 1 \). Plot production function, total cost, average and marginal. Supply function?

• **Case 1b.** \( \alpha = 1 \). Plot production function, total cost, average and marginal. Supply function?

• **Case 1c.** \( \alpha < 1 \). Plot production function, total cost, average and marginal. Supply function?
• **Case 2.** *Non-convex technology.* Plot production function, total cost, average and marginal. Supply function?

• **Case 3.** *Technology with setup cost.* Plot production function, total cost, average and marginal. Supply function?
2.1 Supply Function

- Supply function: \( y^* = y^* (w, r, p) \)

- What happens to \( y^* \) as \( p \) increases?

- Is the supply function upward sloping?

- Remember f.o.c:
  \[ p - c'_y (w, r, y) = 0 \]

- Implicit function:
  \[ \frac{\partial y^*}{\partial p} = - \frac{1}{-c''_{y,y} (w, r, y)} > 0 \]
  as long as s.o.c. is satisfied.

- Yes! Supply function is upward sloping.
3 One-step Profit Maximization

- Nicholson, Ch. 9, pp. 265–270 [OLD: Ch. 13, pp. 346–350].

- One-step procedure: maximize profits

- Perfect competition. Price $p$ is given
  - Firms are small relative to market
  - Firms do not affect market price $p_M$

- Will firm produce at $p > p_M$?
- Will firm produce at $p < p_M$?
  - $\implies p = p_M$
• Revenue: $py = pf(L, K)$

• Cost: $wL + rK$

• Profit $pf(L, K) - wL - rK$
• Agent optimization:

$$\max_{L,K} pf(L, K) - wL - rK$$

• First order conditions:

$$pf'_{L}(L, K) - w = 0$$

and

$$pf'_{K}(L, K) - r = 0$$

• Second order conditions? $$pf''_{L,L}(L, K) < 0$$ and

$$|H| = \begin{vmatrix}
    pf''_{L,L}(L, K) & pf''_{L,K}(L, K) \\
    pf''_{L,K}(L, K) & pf''_{K,K}(L, K)
\end{vmatrix} =$$

$$= p^2 \left[ f''_{L,L}f''_{K,K} - \left( f''_{L,K} \right)^2 \right] > 0$$

• Need $$f''_{L,K}$$ not too large for maximum
• Comparative statics with respect to to $p$, $w$, and $r$.

• What happens if $w$ increases?

$$\frac{\partial L^*}{\partial w} = -\frac{\begin{vmatrix} -1 & p f''_{L,K}(L,K) \\ 0 & p f''_{K,K}(L,K) \end{vmatrix}}{p f''_{L,L}(L,K) \quad p f''_{L,K}(L,K) \quad p f''_{K,L}(L,K) \quad p f''_{K,K}(L,K)} < 0$$

and

$$\frac{\partial L^*}{\partial r} =$$

• Sign of $\frac{\partial L^*}{\partial r}$ depends on $f''_{L,K}$.
4 Introduction to Market Equilibrium


- Two ways to analyze firm behavior:
  - Two-Step Cost Minimization
  - One-Step Profit Maximization

- What did we learn?
  - Optimal demand for inputs $L^*, K^*$ (see above)
  - Optimal quantity produced $y^*$
• **Supply function.** $y = y^* (p, w, r)$

  – From profit maximization:
    $$y = f (L^* (p, w, r), K^* (p, w, r))$$

  – From cost minimization:
    \[ MC \text{ curve above } AC \]

  – Supply function is increasing in $p$

• Market Equilibrium. Equate demand and supply.

• Aggregation?

• Industry supply function!
5  Next Lecture

• Aggregation

• Market Equilibrium

• Comparative Statics of Equilibrium

• Taxes and Subsidies

• Long-Run Equilibrium